Math 222 Practice Problems 3

Sections 5.2, 5.4 & 5.5

1. Five students in a class each count how many friends they have in the class (not including themselves). We assume that if A is friends with B, then B is also friends with A. Show that at least two of the students have the same number of friends.

2. A company has 150 employees. Each employee speaks some of the following languages and no other languages: English, French, Arabic, Mandarin, Cantonese, Spanish. Show that at least three employees speak exactly the same set of languages.

3. Use the binomial theorem to find the coefficient of x^5y^7 in the expansion of $(3x - 8y)^{12}$.

4. Let $k \geq 1$. Use the formula $nCk = \frac{n!}{k!(n-k)!}$ to prove that $k \cdot nCk = n \cdot (n-1)C(k-1)$.

5. Find the number of ways to select 5 objects from a group of 9 if:
a) order matters and repetition is not allowed
b) order matters and repetition is allowed
c) order does not matter and repetition is not allowed
d) order does not matter and repetition is allowed

6. Find the number of integer solutions to the following equation if $x_1, x_3, x_4, x_5 \geq 0, x_2 \geq 4$ and $x_6 \geq 2$: $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 27$.

Sections 7.1 & 7.2

7. a) Find a recurrence relation for the number of bit strings of length $n \geq 2$ that contain two consecutive zeros.
b) How many bit strings of length 8 contain two consecutive zeros?

8. A codeword consists of symbols 0 to 5 and it must have an odd number of zeros.
a) Find a recurrence relation for the number of allowable codewords of length $n \geq 1$.
b) How many allowable codewords of length 5 are there?

9. Solve $a_{n+2} = -4a_{n+1} + 12a_n$ for $n \geq 1$, $a_1 = 16, a_2 = 104$.

10. Solve $a_n = 2a_{n-1} + 1$ for $n \geq 2$, $a_1 = 1$.

Section 7.5

11. Use the inclusion/exclusion formula for three sets to find $|A \cap C|$ if: $|A \cup B \cup C| = 58, |A| = 40, |B| = 31, |C| = 34, |A \cap B| = 20, |B \cap C| = 18$ and $|A \cap B \cap C| = 12$.