Practice Problems 1

(1) If more than 20 lines are in use, then outgoing calls cannot be made.

$q \Rightarrow (\sim p)$

(2)

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Columns are identical.

Yes, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

logically equivalent

(3) 1101 0011 $\oplus$ ((0111 1001 $\wedge$ 0110 0001) $\vee$ 0010 1000)

= 1101 0011 $\oplus$ ((0110 0001) $\vee$ 0010 1000)

= 1101 0011 $\oplus$ (0110 1001)

= 1011 1010

(4) Let's call the expression
$f(p,q,r)$

p	q	r	$f(p,q,r)$
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

$p \wedge q \wedge (\sim r)$

$p \wedge (\sim q) \wedge r$

$(\sim p) \wedge q \wedge r$

$f(p,q,r):\ (p \wedge q \wedge (\sim r)) \vee (p \wedge (\sim q) \wedge r) \vee ((\sim p) \wedge q \wedge r)$

(5) a) True $\quad -3(x-2) = -3x+6 = 6-3x$ for all real x

b) True $\quad$ for example, $x=0$

c) False $\quad 0.1^3 \neq 0.1$

d) True $\quad$ for example, $x=0$

e) False $\quad 0^2 \neq -5$

f) False $\quad$ no x-value works

(6) a) False when $x=0$ no y-value works

b) True $x=0$ $\forall y[0=0]$ is true

c) True $x=-1$ $y=1$ $[-(-1)^3=1^4]$ is true

d) False when $x=1$ no y-value works
(y must be an integer).

(7) a) n is odd.

$\Rightarrow n=2k+1$, for some integer k

$\Rightarrow n^2+3n+7=(2k+1)^2+3(2k+1)+7$

$\Rightarrow n^2+3n+7=(4k^2+4k+1)+6k+3+7$

$\Rightarrow n^2+3n+7=4k^2+10k+11$

$\Rightarrow n^2+3n+7=2(2k^2+5k+5)+1$, where $2k^2+5k+5$ is an integer

$\Rightarrow n^2+3n+7$ is odd.

b) Contrapositive: If n is even, then n^3+5n^2+n is even.
$(\sim q \Rightarrow \sim p)$

Proof: n is even

$\Rightarrow n=2k$, for some integer k

$\Rightarrow n^3+5n^2+n=(2k)^3+5(2k)^2+2k$

$\Rightarrow n^3+5n^2+n=8k^3+5(4k^2)+2k$

$\Rightarrow n^3+5n^2+n=8k^3+20k^2+2k$

$\Rightarrow n^3+5n^2+n=2(4k^3+10k^2+k)$, where $4k^3+10k^2+k$ is an integer

$\Rightarrow n^3+5n^2+n$ is even.

(8) Proof:

Case I. $z = 3k$, where k is an integer.

$$z = 5(0) + 3(k)$$

Case II. $z = 3k+1$, where k is an integer

Notice $1 = 5(-1) + 3(2)$

So $3k+1 = 5(-1) + 3(k+2)$

$$z = 5(-1) + 3(k+2) \quad (k+2 \text{ is an integer})$$

Case III. $z = 3k+2$, where k is an integer

Notice $2 = 5(1) + 3(-1)$

So $3k+2 = 5(1) + 3(k-1)$

$$z = 5(1) + 3(k-1) \quad (k-1 \text{ is an integer})$$

(9) a) $A \times B = \{(a,-1), (a,0), (a,1), (b,-1), (b,0), (b,1)\}$.

b) $P(B) = \{\{-1,0,1\}, \{-1,0\}, \{-1,1\}, \{0,1\}, \{-1\}, \{0\}, \{1\}, \emptyset\}$

c) $|P(A)| = 2^{|A|}$ for any set A.

$|B \times B| = |B| \cdot |B| = 9$

So $|P(B \times B)| = 2^9 = 512$

(10) Let $x \in \overline{A} \cap \overline{B}$

$\Rightarrow$ $x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow$ $x \notin A$ and $x \notin B$

$\Rightarrow$ $x \notin A \cup B$

$\Rightarrow$ $x \in \overline{A \cup B}$

(11) $f(7) = \lfloor \lfloor \lceil 7+\pi \rceil - e \rfloor - 10\sqrt{2} \rfloor$

$= \lfloor \lfloor 11 - e \rfloor - 10\sqrt{2} \rfloor$

$= \lfloor 8 - 10\sqrt{2} \rfloor$

$= -7$

(12) a) $f(a)$ can be 3, 4, or 5
and $f(b)$ can be 3, 4, or 5
functions $= 3 \times 3 = 9$

b) For the function to be one-to-one
$f(a)$ and $f(b)$ must be different

This rules out 3 functions: $\begin{cases} f(a)=3 \\ f(b)=3 \end{cases}$ $\begin{cases} f(a)=4 \\ f(b)=4 \end{cases}$ $\begin{cases} f(a)=5 \\ f(b)=5 \end{cases}$

$9 - 3 = 6$

[Alternatively: # ways to choose $f(x)$
$= 3$ ways to choose $f(a)$
$\times 2$ ways to choose $f(b)$
$= 6$]

c) Total # of functions from C to D

= 2 choices for $f(a)$ X 2 choices for $f(b)$ x 2 choices for $f(c)$

= 8

that are not onto = 2

$$\begin{cases} f(a)=1 \\ f(b)=1 \\ f(c)=1 \end{cases} \qquad \begin{cases} f(a)=5 \\ f(b)=5 \\ f(c)=5 \end{cases}$$

onto functions from C to D = $8-2=6$

(13) Show $f(a)=f(b) \Rightarrow a=b$

Proof: Let $f(a)=f(b)$

$\Rightarrow 11a-3 = 11b-3$

$\Rightarrow 11a = 11b$

$\Rightarrow 11a-11b=0$

$\Rightarrow 11(a-b)=0$

$\Rightarrow a-b=0$

$\Rightarrow a=b.$