Math 222 Practice Problems 1

Sections 1.1 and 1.2

1. Express the statement "Outgoing calls cannot be made when more than 20 lines are in use" in terms of logical symbols and the statements:
p: Outgoing calls can be made.
q: More than 20 lines are in use.

2. Use a truth table to confirm the second distributive law:
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

3. Use logical operations on bit strings to simplify:
$1101\ 0011 \oplus ((0111\ 1001 \wedge 0110\ 0001) \vee 0010\ 1000)$.

4. Construct a logical expression using only statements p, q, r and operations $\sim, \vee, \wedge$ that is TRUE whenever exactly two out three of p, q, r are true, and is FALSE otherwise.

Sections 1.3 and 1.4

5. The domain of $P(x), Q(x)$ and $R(x)$ is the set of all real numbers.
$P(x)$: $-3(x-2) = 6 - 3x$
$Q(x)$: $x^3 \geq x$
$R(x)$: $x^2 = -5$
Write TRUE or FALSE for each statement. Briefly justify each answer.
a) $\forall x P(x)$
b) $\exists x P(x)$
c) $\forall x Q(x)$
d) $\exists x Q(x)$
e) $\forall x R(x)$
f) $\exists x R(x)$

6. The domain of each variable is the set of integers. Write TRUE or FALSE for each statement. Briefly justify each answer.
a) $\forall x \exists y \, [x = y^2 + 1]$

b) $\exists x \forall y \, [xy = x^2]$

c) $\exists x \exists y \, [-x^3 = y^4]$

d) $\forall x \exists y \, [3x = 2y]$

Sections 1.6 and 1.7

7. a) Use a direct proof to show that: If n is an odd integer, then $n^2 + 3n + 7$ is an odd integer.
b) Use a proof by contrapositive to show that: If $n^3 + 5n^2 + n$ is an odd integer, then n is an odd integer.

8. Prove that every integer z can be written as $z = 5a + 3b$, where a and b are integers. Use a proof by cases:
Case I: $z = 3k$ where k is an integer.
Case II: $z = 3k + 1$ where k is an integer.
Case III: $z = 3k + 2$ where k is an integer.

Sections 2.1-2.3

9. Let $A = \{a, b\}$ and $B = \{-1, 0, 1\}$.
a) Write out the set $A \times B$.
b) Write out $\mathcal{P}(B)$, the power set of B.
c) Find $|\mathcal{P}(B \times B)|$.

10. Show that for any sets A and B, $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$.

11. Let $f(x) = \lfloor \lfloor \lceil x + \pi \rceil - e \rfloor - 10\sqrt{2} \rfloor$ be a function from the real numbers to the integers. Find $f(7)$.

12. a) How many different functions are there from the set $A = \{a, b\}$ to the set $B = \{3, 4, 5\}$?
b) How many of these are one-to-one?
c) How many onto functions are there from $C = \{a, b, c\}$ to $D = \{1, 5\}$?

13. Show that the function $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(z) = 11z - 3$ is one-to-one.