THE RULES OF INFERENCE

Section 1. Valid Arguments

An **argument** is a sequence of statements that end with a conclusion. An argument is **valid** when the conclusion follows from the previous statements.

Example: A Valid Argument
"You are a Camosun student."
"If you are a Camosun student, then you can log on to Camlink."
Therefore, "You can log on to Camlink."

In logic the initial statements are called **premises**. In logical form the above argument looks like:

p
$p \Rightarrow q$
$\therefore q$

We can confirm that an argument is valid using a truth table. We check that $[p \wedge (p \Rightarrow q)] \Rightarrow q$ is a tautology (always true).

p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$	$[p \wedge (p \Rightarrow q)] \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the last column is all true, the argument is valid.

Section 2. The Rules of Inference

Eight main rules of inference are used as building blocks to construct more complicated arguments. The eight rules are listed here by their formal names.

MODUS PONENS

$$\begin{array}{l} p \\ \underline{p \Rightarrow q} \\ \therefore \quad q \end{array}$$

MODUS TOLLENS

$$\begin{array}{l} \sim q \\ \underline{p \Rightarrow q} \\ \therefore \quad \sim p \end{array}$$

HYPOTHETICAL SYLLOGISM

$$\begin{array}{l} p \Rightarrow q \\ \underline{q \Rightarrow r} \\ \therefore \; p \Rightarrow r \end{array}$$

DISJUNCTIVE SYLLOGISM

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ \therefore \quad q \end{array}$$

ADDITION

$$\begin{array}{l} \underline{p} \\ \therefore p \vee q \end{array}$$

SIMPLIFICATION

$$\begin{array}{l} \underline{p \wedge q} \\ \therefore \quad p \end{array}$$

CONJUNCTION

$$\begin{array}{l} p \\ \underline{q} \\ \therefore \quad p \wedge q \end{array}$$

RESOLUTION

$$\begin{array}{l} \quad p \vee q \\ \underline{\sim p \vee r} \\ \therefore \quad q \vee r \end{array}$$

Example: Let p: "You are sick." Let q: "You stay home from school."
The Modus Tollens rule looks like:

"You don't stay home from school."
"If you are sick, then you stay home from school."
Therefore, "You are not sick."

Now let p: "You go to the gym." Let q: "You play basketball."
The Disjunctive Syllogism rule looks like:

"You go to the gym or you play basketball."
"You don't go to the gym."
Therefore, "You play basketball."

Example: Confirm the Hypothetical Syllogism rule using a truth table.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since the last column is all true, the argument is valid.

Section 3. Building Arguments

When more than three propositions are involved in an argument (say p, q, r and s), it is easier to use the Rules of Inference to check the validity of an argument than to use truth tables. A truth table involving propositions p, q, r and s would have $2^4 = 16$ rows!

Example: Use the Rules of Inference to confirm that the argument below is valid.

$$\begin{array}{l} \sim p \wedge q \\ \quad r \Rightarrow p \\ \sim r \Rightarrow s \\ \underline{\quad s \Rightarrow t \quad} \\ \therefore \quad\quad t \end{array}$$

1. $\sim p \wedge q$	Premise
2. $\sim p$	Simplification of 1.
3. $r \Rightarrow p$	Premise
4. $\sim r$	Modus Tollens using 2. and 3.
5. $\sim r \Rightarrow s$	Premise
6. s	Modus Ponens using 4. and 5.
7. $s \Rightarrow t$	Premise
8. t	Modus Ponens using 6. and 7.

In each line we indicate that the statement is a premise, or can be deduced from previous statements using a Rule of Inference.

Example: Use the Rules of Inference to confirm that the argument below is valid.

$$\begin{array}{l} \quad p \Rightarrow q \\ \sim p \Rightarrow r \\ \underline{\quad r \Rightarrow s \quad} \\ \therefore \quad \sim q \Rightarrow s \end{array}$$

1. $p \Rightarrow q$	Premise
2. $\sim q \Rightarrow \sim p$	Contrapositive of 1.
3. $\sim p \Rightarrow r$	Premise
4. $\sim q \Rightarrow r$	Hypothetical Syllogism using 2. and 3.
5. $r \Rightarrow s$	Premise
6. $\sim q \Rightarrow s$	Hypothetical Syllogism using 4. and 5.

Section 4. Logical Fallacies

A fallacy is an invalid argument, especially one that is easily confused with a valid argument. Below are examples of two common fallacies.

The Fallacy of Affirming the Conclusion

$p \Rightarrow q$

$\underline{q \quad\quad}$

$\therefore \quad p$ INVALID ARGUMENT!

Example: Use a truth table to confirm that the above argument is invalid.

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$[(p \Rightarrow q) \wedge q] \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The argument is invalid because the last column is not all true.

The Fallacy of Denying the Hypothesis

$p \Rightarrow q$

$\underline{\sim p \quad\quad}$

$\therefore \quad \sim q$ INVALID ARGUMENT!

Example: Use a truth table to confirm that the above argument is invalid.

p	q	$p \Rightarrow q$	$\sim p$	$(p \Rightarrow q) \wedge (\sim p)$	$\sim q$	$[(p \Rightarrow q) \wedge (\sim p)] \Rightarrow \sim q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

The argument is invalid because the last column is not all true.

Section 5. Exercises

1. Confirm the Addition rule using a truth table.

2. Confirm the Conjunction rule using a truth table.

3. Confirm the Resolution rule using a truth table.

4. Use the Rules of Inference to confirm that the argument below is valid.

$$\begin{array}{l} \sim p \Rightarrow q \\ \sim r \Rightarrow q \\ \sim q \\ \underline{(p \wedge r) \Rightarrow s} \\ \therefore \quad s \end{array}$$

5. Use the Rules of Inference to confirm that the argument below is valid.

$$\begin{array}{l} p \vee q \\ \sim q \\ p \Rightarrow \sim r \\ r \vee s \\ \underline{\sim t \Rightarrow \sim s} \\ \therefore \quad t \end{array}$$

6. State whether each of the following arguments is valid or invalid. Just read each argument carefully–you shouldn't have to do any work.

a) "All students in Math 222 love logic."
"Dave is in Math 222."
Therefore, "Dave loves logic."

b) "All bridge students are sleep-deprived."
"Stan is sleep-deprived."
Therefore, "Stan is a bridge student."

c) "All Canadians eat Cheerios."
"Al is not a Canadian."
Therefore, "Al does not eat Cheerios."

d) "Victoria is in British Columbia."
"Bob does not live in British Columbia."
Therefore, "Bob does not live in Victoria."