

Math 222 Assignment Three

Name: _____

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [3 marks] Sixteen students take a quiz. The possible marks are the integers 0 – 15. If the sixteen marks sum to 100, prove that at least two students got the same mark.

2. [4 marks] Let m, r and j be nonnegative integers. Use the formula $nCr = \frac{n!}{k!(n-k)!}$ to prove that $mCr \cdot rCj = mCj \cdot (m-j)C(r-j)$.

3. [5 marks] a) State the Binomial Theorem.

b) Use part a) to find a simpler expression for $\sum_{k=0}^n \binom{n}{k} 2^k$.

c) Confirm your answer to part b) using the value $n = 6$.

4. [4 marks] Find the number of integer solutions to the following equation if $x_1, x_3, x_5 \geq 0, x_2 \geq -5$ and $x_4 \geq 2$: $x_1 + x_2 + x_3 + x_4 + x_5 = 21$.

5. [5 marks] We are rolling a four-sided die (with sides labelled 1, 2, 3, 4). Find a recurrence relation for the number of ways the rolls can sum to n for $n \geq 1$. (For example, here are the ways in which the rolls can sum to 3: 1-1-1, 1-2, 2-1, 3).

6. [6 marks] Solve the following recurrence relation:
 $a_n = 10a_{n-1} - 25a_{n-2} + 4 \cdot 3^n$ for $n \geq 2$, $a_0 = 11$, $a_1 = 17$.

7. [3 marks] a) State the inclusion/exclusion formula for three sets A, B, C .

b) Use the formula to find $|A \cup B|$ if $|A \cup B \cup C| = 80$, $|A| = 56$,
 $|B| = 57$, $|C| = 59$, $|A \cap C| = 46$, $|B \cap C| = 44$ and $|A \cap B \cap C| = 41$.