

Math 222 Assignment Two

Name: _____

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [3 marks] Show that $(3n + 7) \log_2(n^2 + 1)$ is $O(n \log_2 n)$.

2. [4 marks] a) Write 4599504 as a product of prime powers.
(For example $2646 = 2 \times 3^3 \times 7^2$).

b) Write 907500 as a product of prime powers.

c) Write $\gcd(4599504, 907500)$ as a product of prime powers.

3. [3 marks] Compute $9^{39} \pmod{11}$ using successive squaring. Show all your work.

4. [6 marks] a) Use the Euclidean Algorithm to find $\gcd(1545, 240)$.

b) Use part a) to find integers x and y so that $165 = 1545x + 240y$.

5. [4 marks] Use mathematical induction to show that $1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$ for $n \geq 1$.

6. [4 marks] Use mathematical induction to show that $(1 - \frac{1}{2})(1 - \frac{1}{3}) \cdot \dots \cdot (1 - \frac{1}{n+1}) = \frac{1}{n+1}$ for $n \geq 1$.

7. [6 marks] Give a recursive definition for each sequence below.

a) $a_n = 3(5n + 1) - 8$ for $n \geq 0$

b) the sequence $a_1, a_2, a_3, a_4, \dots$ given by $-3, 4, -3, 4, \dots$

c) the sequence $a_1, a_2, a_3, a_4, a_5, a_6, \dots$ given by $1, 3, 7, 15, 31, 63, \dots$