Math 222 Assignment One

Name:________________________________

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [4 marks] Use a truth table to decide whether the following two statements are logically equivalent: $p \vee (q \oplus r)$ and $((p \vee q) \oplus (p \vee r))$. Show all your work and explain your answer.

p	q	r	$q \oplus r$	$p \vee (q \oplus r)$	$p \vee q$	$p \vee r$	$(p \vee q) \oplus (p \vee r)$
T	T	T	F	T	T	T	F
T	T	F	T	T	T	T	F
T	F	T	T	T	T	T	F
T	F	F	F	T	T	T	F
F	T	T	F	F	T	T	F
F	T	F	T	T	T	F	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	F

Columns are different
They are not logically equivalent

2. [4 marks] Find an expression logically equivalent to $f(p, q, r)$ using only statements p, q, r and operations $\sim, \vee, \wedge$:

p	q	r	$f(p,q,r)$	
T	T	T	F	
T	T	F	T	$p \wedge q \wedge (\sim r)$
T	F	T	F	
T	F	F	F	
F	T	T	F	
F	T	F	T	$(\sim p) \wedge q \wedge (\sim r)$
F	F	T	T	$(\sim p) \wedge (\sim q) \wedge r$
F	F	F	F	

$$(p \wedge q \wedge (\sim r)) \vee ((\sim p) \wedge q \wedge (\sim r)) \vee ((\sim p) \wedge (\sim q) \wedge r)$$

OR $$(q \wedge (\sim r)) \vee ((\sim p) \wedge (\sim q) \wedge r)$$

3. [4 marks] Determine the truth values of each of these statements if the domain of each variable consists of all real numbers.

a) $\forall x\ \exists y(x^2 = y)$

True For all x $y = x^2$ is a real number.

b) $\forall x\ \exists y(x = y^2)$

False When $x = -1$, no y-value works

c) $\exists x\ \forall y(y \neq 0 \rightarrow xy = 1)$

False No single x-value works for all y-values.

d) $\forall x\ \exists y(x + y = 2 \wedge 2x - y = 1)$

False when $x = 0$, no y-value works

4. [3 marks] Let n be an integer. Prove that if n^2+6 is odd then $1-n$ is even.

Contrapositive: If $1-n$ is odd then n^2+6 is even.

$(\sim q \Rightarrow \sim p)$

Proof: Let $1-n$ be an odd number

$\Rightarrow \quad 1-n = 2k+1$ for some integer k

$\Rightarrow \quad -n = 2k$

$\Rightarrow \quad n = -2k$

$\Rightarrow \quad n^2+6 = (-2k)^2+6$

$\Rightarrow \quad n^2+6 = 4k^2+6$

$\Rightarrow \quad n^2+6 = 2(2k^2+3)$, where $2k^2+3$ is an integer

$\Rightarrow \quad n^2+6$ is even.

5. [3 marks] We know that $(a-b)^2 \geq 0$ for any real numbers a and b. Use this fact to prove that $x^2 + \frac{1}{x^2} \geq 2$ for any nonzero real number x.

Rough work:
Work backwards

$x^2 + \frac{1}{x^2} - 2 \geqslant 0$

$x^2 - 2 + \frac{1}{x^2} \geqslant 0$

$(x - \frac{1}{x})^2 \geqslant 0$

Proof: $(x - \frac{1}{x})^2 \geqslant 0$ for any nonzero x

$\Rightarrow x^2 - 2x\frac{1}{x} + (\frac{1}{x})^2 \geqslant 0$

$\Rightarrow x^2 - 2 + \frac{1}{x^2} \geqslant 0$

$\Rightarrow x^2 + \frac{1}{x^2} \geqslant 2$.

6. [4 marks] Let $f : \{1,2,3,4,5,6,7\} \to \mathbb{Z}$, given by:
$f(1) = 2, f(2) = 1, f(3) = -1, f(4) = 2, f(5) = 2, f(6) = -1, f(7) = 3$.

a) State the domain of f.

domain$(f) = \{1,2,3,4,5,6,7\}$

b) State the codomain of f.

Codomain$(f) = \mathbb{Z}$, the set of integers

c) State the range of f.

range$(f) = \{2, 1, -1, 3\}$

d) Find the preimage(s) of 2.

$f(1) = 2 \quad f(4) = 2 \quad f(5) = 2$

Preimages of 2 are 1, 4 and 5

7. [3 marks] Describe the set of real numbers x so that $\lfloor \lceil x + 0.5 \rceil - \frac{8}{3} \rfloor = -6$.

$\Rightarrow \lfloor \lceil x + 0.5 \rceil - 2.\bar{6} \rfloor = -6$

$\Rightarrow \lceil x + 0.5 \rceil = -3$

$\Rightarrow -4 < x + 0.5 \leq -3$

$\Rightarrow -4.5 < x \leq -3.5$

or $(-4.5, -3.5]$

8. [5 marks] Each function below is from the set of integers to the set of integers. State whether each function is one-to-one, onto, both or neither.

a) $f(x) = x^3 - 1$

one-to-one, not onto

b) $f(x) = x^4 + 1$

neither

c) $f(x) = 7x - 5$

one-to-one, not onto

d) $f(x) = x - 6$

both one-to-one and onto

e) $f(x) = \lfloor \frac{x}{2} \rfloor$

onto, not one-to-one

Explanation:

a) $f(-2) = -7$
$f(-1) = 0$ one-to-one
$f(0) = 1$
$f(1) = 2$
$\vdots$

not onto
$f(x) \neq 3$

b) $f(1) = 2$
$f(-1) = 2$
not one-to-one

not onto
$f(x) \neq 0$

c) $f(-1) = -12$
$f(0) = -5$ one-to-one
$f(1) = 2$

not onto
$f(x) \neq 0$

d) $f(-1) = -7$
$f(0) = 6$
$f(1) = 5$
one-to-one and onto

e) $f(-2) = -1$
$f(-1) = \lfloor -0.5 \rfloor = -1$
$f(0) = 0$
$f(1) = 0$
$f(2) = 1$
] not one-to-one

onto ✓