

## Math 222 Assignment One

Name: \_\_\_\_\_

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [4 marks] Use a truth table to decide whether the following two statements are logically equivalent:  $p \vee (q \oplus r)$  and  $((p \vee q) \oplus (p \vee r))$ . Show all your work and explain your answer.

2. [4 marks] Find an expression logically equivalent to  $f(p, q, r)$  using only statements  $p, q, r$  and operations  $\sim, \vee, \wedge$ :

$p$	$q$	$r$	$f(p, q, r)$
$T$	$T$	$T$	$F$
$T$	$T$	$F$	$T$
$T$	$F$	$T$	$F$
$T$	$F$	$F$	$F$
$F$	$T$	$T$	$F$
$F$	$T$	$F$	$T$
$F$	$F$	$T$	$T$
$F$	$F$	$F$	$F$

3. [4 marks] Determine the truth values of each of these statements if the domain of each variable consists of all real numbers.

a)  $\forall x \exists y(x^2 = y)$

b)  $\forall x \exists y(x = y^2)$

c)  $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$

d)  $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$

4. [3 marks] Let  $n$  be an integer. Prove that if  $n^2 + 6$  is odd then  $1 - n$  is even.

5. [3 marks] We know that  $(a - b)^2 \geq 0$  for any real numbers  $a$  and  $b$ . Use this fact to prove that  $x^2 + \frac{1}{x^2} \geq 2$  for any nonzero real number  $x$ .

6. [4 marks] Let  $f : \{1, 2, 3, 4, 5, 6, 7\} \rightarrow \mathbb{Z}$ , given by:  
 $f(1) = 2, f(2) = 1, f(3) = -1, f(4) = 2, f(5) = 2, f(6) = -1, f(7) = 3$ .

a) State the domain of  $f$ .

b) State the codomain of  $f$ .

c) State the range of  $f$ .

d) Find the preimage(s) of 2.

7. [3 marks] Describe the set of real numbers  $x$  so that  $\lfloor [x + 0.5] - \frac{8}{3} \rfloor = -6$ .

8. [5 marks] Each function below is from the set of integers to the set of integers. State whether each function is one-to-one, onto, both or neither.

a)  $f(x) = x^3 + 1$

b)  $f(x) = x^4 + 1$

c)  $f(x) = 7x - 5$

d)  $f(x) = x - 6$

e)  $f(x) = \lfloor \frac{x}{2} \rfloor$