

Quiz

$$y'' - 4y' + 7y = 0$$

$$m^2 - 4m + 7 = 0$$

$$m = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 7}}{2}$$

$$m = \frac{4 \pm \sqrt{-12}}{2} \leftarrow \sqrt{4} \sqrt{3} \sqrt{-1} = 2\sqrt{3}j$$

$$m = \frac{4 \pm 2\sqrt{3}j}{2}$$

$$m = 2 \pm \sqrt{3}j$$

$$m = \alpha \pm \beta j \quad \alpha = 2 \quad \beta = \sqrt{3}$$

$$y = e^{\alpha x} (C_1 \cos \beta x + C_2 \sin \beta x)$$

$$y = e^{2x} (C_1 \cos \sqrt{3}x + C_2 \sin \sqrt{3}x)$$

Quiz Tues 12th 31.10

Test Thurs 7th

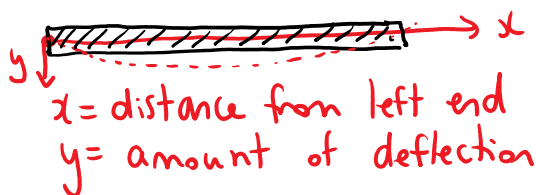
Omit 31.8 # 29, 31

31.9 # 15, 29

31.10 # 7

31.10 Cont'd

Deflection of a Beam (Statics)



$$EI \frac{d^4 y}{dx^4} = w(x)$$

↑ stiffness of beam (constant)
 ↑ load

Formula Sheet

Ex: Beam of length L has constant w
(due to its own weight). Find the deflection y .

At $x=0$: $y=0$ and $y''=0$

At $x=L$: $y=0$ and $y''=0$

$$EI \frac{d^4 y}{dx^4} = w$$

$$\frac{d^4 y}{dx^4} = \left(\frac{w}{EI} \right) \leftarrow \text{Call this } k$$

$$y'''' = k$$

Integrate (because DE is quite simple)
with respect to x

$$y''' = kx + C_1$$

$$y'' = \frac{kx^2}{2} + C_1x + C_2$$

$$y' = \frac{kx^3}{6} + \frac{C_1x^2}{2} + C_2x + C_3$$

$$y = \frac{kx^4}{24} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$

Recall

$$x=0: y=0, y''=0$$

$$x=L: y=0, y''=0$$

$$x=0: \boxed{0 = C_4}$$

$$x=0: y'' = \frac{kx^2}{2} + C_1x + C_2$$

$$y''=0: \boxed{0 = C_2}$$

$$x=L: \underline{0 = \frac{kL^4}{24} + \frac{C_1L^3}{6} + C_3L} \quad (1)$$

$$y=0: (C_2=0=C_4)$$

$$x=L \rightarrow y'' = \frac{kx^2}{2} + C_1x + C_2$$

$$\underline{0 = \frac{kL^2}{2} + C_1L} \quad (2)$$

$$(C_2=0)$$

$$0 = \frac{kL^2}{2} + C_1 L \quad (C_2 = 0) \quad (2)$$

Solve (2) for C_1 :

$$-\frac{kL^2}{2} = C_1 L$$

$$\boxed{-\frac{kL}{2} = C_1}$$

$$C_1 = -\frac{kL}{2} \rightarrow (1)$$

$$0 = \frac{kL^4}{24} + \left(-\frac{kL}{2}\right) \frac{L^3}{6} + C_3 L$$

$$0 = \frac{kL^4}{24} - \frac{2kL^4}{24} + C_3 L$$

$$0 = -\frac{kL^4}{24} + C_3 L$$

$$\frac{kL^4}{24} = C_3 L$$

$$\boxed{\frac{kL^3}{24} = C_3}$$

Recall $y = \frac{kx^4}{24} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$
 $(C_2 = 0) \quad (C_4 = 0)$

$$y = \frac{kx^4}{24} + \frac{2}{2} \left(-\frac{kL}{2}\right) \frac{x^3}{6} + \left(\frac{kL^3}{24}\right) x$$

$$y = \frac{kx^4}{24} - \frac{2kLx^3}{24} + \frac{kL^3x}{24}$$

$$y = \frac{k}{24} (x^4 - 2Lx^3 + L^3x)$$

Recall $k = \frac{w}{EI}$

$$y = \frac{w}{24EI} (x^4 - 2Lx^3 + L^3x)$$