Week 9 Tuesday

March 5, 2019 7:54 AM

Quiz $y'' - 4y' + 7y = 0$

$m^2 - 4m + 7 = 0$

$m = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 7}}{2}$

$m = \frac{4 \pm \sqrt{-12}}{2}$ ← $\sqrt{4}\sqrt{3}\sqrt{-1} = 2\sqrt{3}j$

$m = \frac{4 \pm 2\sqrt{3}j}{2}$

$m = 2 \pm \sqrt{3}j$

$m = \alpha \pm \beta j \qquad \alpha = 2 \quad \beta = \sqrt{3}$

$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

$y = e^{2x}(C_1 \cos \sqrt{3}x + C_2 \sin \sqrt{3}x)$

Quiz Tues 12th 31.10

Test Thurs 7th

Omit 31.8 # 29, 31
31.9 # 15, 29
31.10 # 7

31.10 Cont'd

Deflection of a Beam (Statics)

x = distance from left end
y = amount of deflection

$$EI\frac{d^4y}{dx^4} = w(x)$$ Formula Sheet

EI: stiffness of beam (constant)

$w(x)$: load

Ex: Beam of length L has constant w (due to its own weight). Find the deflection y.

At $x=0$: $y=0$ and $y''=0$

At $x=L$: $y=0$ and $y''=0$

$$EI\frac{d^4y}{dx^4} = w$$

$$\frac{d^4y}{dx^4} = \frac{w}{EI} \leftarrow \text{Call this } k$$

$$y'''' = k$$

Integrate (because DE is quite simple) with respect to x

$$y''' = kx + C_1$$

$$y'' = \frac{kx^2}{2} + C_1x + C_2$$

$$y' = \frac{kx^3}{6} + \frac{C_1x^2}{2} + C_2x + C_3$$

$$y = \frac{kx^4}{24} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$

Recall

$x=0$: $y=0$, $y''=0$

$x=L$: $y=0$, $y''=0$

$x=0$, $y=0$: $\boxed{0 = C_4}$

$x=0$, $y''=0$: $y'' = \frac{kx^2}{2} + C_1x + C_2$

$\boxed{0 = C_2}$

$x=L$, $y=0$: $0 = \frac{kL^4}{24} + \frac{C_1L^3}{6} + C_3L$ (1)

$(C_2 = 0 = C_4)$

$x=L$, $y''=0$ $\rightarrow$ $y'' = \frac{kx^2}{2} + C_1x + C_2$

$0 = \frac{kL^2}{2} + C_1L$ (2)

$(C_2 = 0)$

$$0 = \frac{kL^2}{2} + C_1 L \qquad (C_2 = 0) \qquad (2)$$

Solve (2) for C_1:

$$-\frac{kL^2}{2} = C_1 L$$

$$\boxed{-\frac{kL}{2} = C_1}$$

$C_1 = -\frac{kL}{2} \rightarrow (1)$

$$0 = \frac{kL^4}{24} + \left(-\frac{kL}{2}\right)\frac{L^3}{6} + C_3 L$$

$$0 = \frac{kL^4}{24} - \frac{2kL^4}{24} + C_3 L$$

$$0 = -\frac{kL^4}{24} + C_3 L$$

$$\frac{kL^4}{24} = C_3 L$$

$$\boxed{\frac{kL^3}{24} = C_3}$$

Recall $y = \frac{kx^4}{24} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$

$C_2 = 0$ $\qquad$ $C_4 = 0$

$$y = \frac{kx^4}{24} + \frac{2}{2}\left(\frac{-kL}{2}\right)\frac{x^3}{6} + \left(\frac{kL^3}{24}\right)x$$

$$y = \frac{kx^4}{24} - \frac{2kLx^3}{24} + \frac{kL^3x}{24}$$

$$y = \frac{k}{24}\left(x^4 - 2Lx^3 + L^3x\right)$$

Recall $k = \frac{w}{EI}$

$$y = \frac{w}{24EI}\left(x^4 - 2Lx^3 + L^3x\right)$$