Quiz tomorrow 31.8

Test Thursday
31.1, 31.2, 31.4, 31.6-9   (7 Questions)
• Do Sugg HW
• Practice Problems   www.leahhoward.com

Omit   31.10 #7

31.10 Cont'd

$x < 0$
$x = 0$
$x > 0$

Ex: m = 1kg
Air resistance $\beta = 2 N/(m/s)$
Spring constant   k = 4 N/m
No external force
Find equation of motion.

$$ma = -\beta v - kx + f(t)$$

Recall   x = displacement
$a = x''$
$v = x'$

$$mx'' = -\beta x' - kx + f(t) \longleftarrow \text{external force}$$

$$1x'' = -2x' - 4x + 0$$

$$x'' + 2x' + 4x = 0$$

auxiliary equation $\cancel{m^2 + 2m + 4 = 0}$

$$n^2 + 2n + 4 = 0$$

to avoid confusion with mass

$$n = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 4}}{2} \rightarrow \alpha \pm \beta j$$

$$n = \frac{-2 \pm \boxed{\sqrt{-12}}}{2} = \sqrt{4}\sqrt{3}\sqrt{-1} = 2\sqrt{3} j$$

$$n = \frac{-2 \pm 2\sqrt{3} j}{2}$$

$$n = -1 \pm \sqrt{3} j$$

$\alpha = -1$
$\beta = \sqrt{3}$

$$y = e^{\alpha x} (\qquad)$$

Want displacement x in terms of time t

$$x = e^{\alpha t}(C_1 \cos \beta t + C_2 \sin \beta t)$$

$$\boxed{x = e^{-t}(C_1 \cos \sqrt{3} t + C_2 \sin \sqrt{3} t)}$$

Ex: A 49N weight stretches a spring ⟨2.0 cm.⟩ 0.02 m
The spring is pulled 20cm longer than its
equilibrium length and released.
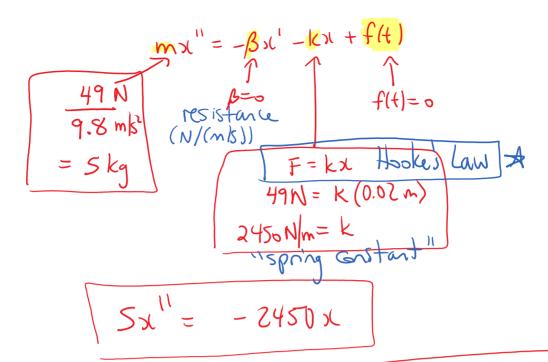There is no damping (no resistance) and
        there is no external force.
    Find the equation of motion.

Formula Sheet $$ma = -\beta v - kx + f(t)$$

$$\boxed{\begin{array}{l} a = x'' \\ v = x' \end{array}}$$

$$mx'' = -\beta x' - kx + f(t)$$

$\dfrac{49 \text{ N}}{9.8 \text{ m/s}^2}$

$= 5 \text{ kg}$

$\beta = 0$
resistance
$(N/(m/s))$

$f(t) = 0$

$F = kx$ Hooke's Law ☆

$49N = k(0.02 m)$

$2450 \text{ N/m} = k$

"spring constant"

$$5x'' = -2450x$$

Initial Conditions
when $t = 0$:
$x = 0.2 \text{ m}$ $x' = 0 \text{ m/s}$
(m)   (m/s)

$$5x'' = -2450x$$

auxiliary equation: $\cancel{5n^2 = -2450}$

$5n^2 = -2450$

$n^2 = -490$

$n = \pm \boxed{\sqrt{-490}} = \sqrt{490}\,\sqrt{-1}$

$n = \pm \sqrt{490}\, j$

or $n \approx \pm 22 j$

$\alpha = 0$
$\beta = 22$

$y = e^{\alpha x} \left(\right)$

$$x = e^{\alpha t}(C_1 \cos \beta t + C_2 \sin \beta t)$$

$$\boxed{x = C_1 \cos 22t + C_2 \sin 22t}$$

$x = 0.2$
$t = 0$:

$$0.2 = C_1 (1) + C_2 (0)$$

$$C_1 = 0.2$$

$$\boxed{x = 0.2 \cos 22t + C_2 \sin 22t}$$

$$x' = 0.2(-22 \sin 22t) + 22 C_2 \cos 22t$$

$x' = 0$
$t = 0$:

$$0 = 0 + 22 C_2 (1)$$

$$C_2 = 0$$

$$\boxed{x = 0.2 \cos 22t}$$

No air resistance
No external force $\Longrightarrow$ Harmonic Motion