Week 8 Friday

March 1, 2019 7:33 AM

31.9 Nonhomogeneous DE Cont'd

Ex: Solve $y'' + 25y = 4e^{-3x}$ if $y' = 2$ and $y = 1$ when $x = 0$

1) Find y_c

$$y'' + 25y = 0$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm\sqrt{-25} \quad \sqrt{25}\sqrt{-1} \rightarrow m = \alpha \pm \beta j$$

$$m = \pm 5j$$

$$m = 0 \pm 5j$$

$\alpha = 0, \beta = 5$: $y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

$$y = C_1 \cos 5x + C_2 \sin 5x$$

y_c

2) Find y_p

$$f(x) = 4e^{-3x}$$

$$y_p = Ae^{-3x}$$

3) $y_p \rightarrow$ DE (Find A)

$$\begin{cases} y_p = Ae^{-3x} \\ y_p' = -3Ae^{-3x} \\ y_p'' = 9Ae^{-3x} \end{cases}$$

DE: $y'' + 25y = 4e^{-3x}$

$$9Ae^{-3x} + 25(Ae^{-3x}) = 4e^{-3x}$$

$$\boxed{9A + 25A}\, e^{-3x} = 4e^{-3x}$$

$9A + 25A = 34A$ $e^{-3x} = 4e^{-3x}$

$34A = 4$

$A = \frac{4}{34} = \frac{2}{17}$

$y_p = Ae^{-3x}$

$y_p = \frac{2}{17}e^{-3x}$

4) $y = y_c + y_p$

$$y = C_1 \cos 5x + C_2 \sin 5x + \frac{2}{17}e^{-3x}$$

5) If there are initial conditions, use them at the end

Recall $y' = 2$ and $y = 1$ when $x = 0$

$y = 1$, $x = 0$: $1 = C_1(1) + C_2(0) + \frac{2}{17}(1)$

$\frac{17}{17} = C_1 + \frac{2}{17}$

$\frac{15}{17} = C_1$

$$y = \frac{15}{17}\cos 5x + C_2 \sin 5x + \frac{2}{17}e^{-3x}$$

Calculate $y' = -5\left(\frac{15}{17}\right)\sin 5x + 5C_2 \cos 5x - \frac{6}{17}e^{-3x}$

$y' = 2$, $x = 0$: $2 = 0 + 5C_2(1) - \frac{6}{17}(1)$

$\frac{34}{17} = 5C_2 - \frac{6}{17}$

$\frac{40}{17} = 5C_2$

$$\frac{40}{17} = 5C_2$$

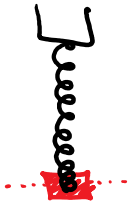
$$\frac{8}{17} = C_2 \rightarrow \text{plug into } y$$

$$y = \frac{15}{17}\cos 5x + \frac{8}{17}\sin 5x + \frac{2}{17}e^{-3x}$$

31.10 Applications : Spring-Mass Systems

Spring-Mass System

Vertical Motion



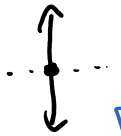
equilibrium position

$F_g = F_{spring}$

Variables :

x = displacement (m)

t = time (s)



spring is compressed $x<0$

eq. position $x=0$

spring is stretched $x>0$

Mass · Acceleration = Net Force

$ma = -\beta v - kx + f(t)$ Formula Sheet

damping force $\propto$ velocity, in opposite direction of motion

e.g. air resistance

restorative force from spring $\propto$ displacement, in opposite direction

any external force

m, β, k : positive constants

Recall $a = x''$

$v = x'$

As a DE : $mx'' = -\beta x' - kx + f(t)$ ★ Know this

Ex: $m = 1\text{kg}$ $\beta = 2\ \text{N/(m/s)}$ $k = 4\ \text{N/m}$ $f(t) = 0$

Find a formula for x.

Rephrased: Find the displacement at any moment in time.

Formula Sheet: $ma = -\beta v - kx + f(t)$

DE : $mx'' = -\beta x' - kx + f(t)$

$x'' = -2x' - 4x$

$x'' + 2x' + 4x = 0$

(Like $y'' + 2y' + 4y = 0$ but with x)