Week 8 Tuesday

February 26, 2019 7:37 AM

Quiz

$$dy = \frac{1}{x} y\,dx + x^4\,dx$$

1) Standard Form

$$dy - \frac{1}{x} y\,dx = x^4\,dx$$

2) I.F.

$$P(x) = -\frac{1}{x}$$

$$\int P(x)\,dx = -\ln x$$

$$\text{I.F.} = e^{\int P(x)\,dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1}$$

3) Multiply Standard Form by I.F.

$$x^{-1}\,dy - x^{-2} y\,dx = x^3\,dx$$

$$d(x^{-1} y) = x^3\,dx$$

4) Integrate

$$x^{-1} y = \frac{x^4}{4} + C$$

Quiz Tues March 5th 31.8

Omit 31.8 # 29, 31

31.8 Repeated or Complex Roots Cont'd

31.8 Repeated or Complex Roots

Complex Numbers

$j = \sqrt{-1}$ (sometimes called i)

$\sqrt{-4} = \sqrt{4}\sqrt{-1} = 2j$

$\sqrt{-7} = \sqrt{7}\sqrt{-1} = \sqrt{7}j$

If roots of the auxiliary equation are $m = \alpha \pm \beta j$ then

$$y = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$$ ← Formula sheet

EX: Solve $D^2y + 2Dy + 2y = 0$

$$m^2 + 2m + 2 = 0$$

$$m = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 2}}{2}$$

$$m = \frac{-2 \pm \sqrt{-4}}{2} \leftarrow \sqrt{4}\sqrt{-1} = 2j$$

$$m = \frac{-2 \pm 2j}{2}$$

$$m = -1 \pm j$$

$$m = -1 \pm j$$

$m = \alpha \pm \beta j \qquad \alpha = -1 \quad \beta = 1$

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

$$y = e^{-x}(C_1 \cos x + C_2 \sin x)$$

Ex: Solve $y'' + y' = -y$

$$y'' + y' + y = 0$$

$$m^2 + m + 1 = 0$$

$$m = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2}$$

$$m = \frac{-1 \pm \sqrt{-3}}{2} \leftarrow \sqrt{3}\sqrt{-1} = \sqrt{3}j$$

$\rightarrow m = \alpha \pm \beta j$

$$m = \frac{-1 \pm \sqrt{3}j}{2}$$

$$m = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j$$

$$\alpha = -\frac{1}{2} \qquad \beta = \frac{\sqrt{3}}{2}$$

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

$$y = e^{\alpha x}\left(C_1 \cos \beta x + C_2 \sin \beta x\right)$$

$$y = e^{-\frac{1}{2}x}\left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x\right)$$