Week 6 Friday

February 15, 2019 7:43 AM

31.7 Higher-Order Homogeneous DE with Constant Coefficients

Notation: $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + y = f(x)$

$y''' - 4y'' + 5y' + y = f(x)$

$D^3y - 4D^2y + 5Dy + y = f(x)$

all mean exactly the same thing

A DE is homogeneous if $f(x) = 0$

non-homogeneous " $f(x) \neq 0$

Quick Ex: Rewrite $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = e^x$

$y'' + 5y' = e^x$ OR $D^2y + 5Dy = e^x$

Is it homogeneous?

NO

Ex: Solve $y'' - 7y' + 12y = 0$

"auxiliary equation"

$$m^2 - 7m + 12 = 0$$
$$(m-3)(m-4) = 0$$
$$m = 3, 4$$

$$\boxed{\begin{array}{l}\text{General Solution to DE:} \\ y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + \ldots + C_n e^{m_n x}\end{array}}$$

Here $y = C_1 e^{3x} + C_2 e^{4x}$ ✓

Alternatively $y = C_1 e^{4x} + C_2 e^{3x}$ ✓

Why does this work?

<u>Check</u>:

$$\begin{cases} y = C_1 e^{3x} + C_2 e^{4x} \\ y' = 3C_1 e^{3x} + 4C_2 e^{4x} \\ y'' = 9C_1 e^{3x} + 16C_2 e^{4x} \end{cases}$$

$$\begin{aligned} \text{LS of DE} &= y'' - 7y' + 12y \\ &= (9C_1 e^{3x} + 16C_2 e^{4x}) - 7(3C_1 e^{3x} + 4C_2 e^{4x}) \\ &\quad + 12(C_1 e^{3x} + C_2 e^{4x}) \\ &= 9C_1 e^{3x} + 16C_2 e^{4x} - 21C_1 e^{3x} - 28C_2 e^{4x} \end{aligned}$$

$$+12C_1e^{3x} + 12C_2e^{4x}$$

$$= \boxed{0}C_1e^{3x} + \boxed{0}C_2e^{4x}$$

$$= 0$$

$$= \text{RS of DE} \checkmark$$

Auxiliary equation guarantees that

LS of DE = 0

$y = C_1e^{m_1x} + C_2e^{m_2x} + \ldots + C_ne^{m_nx}$ works when:

DE is homogeneous

Roots of auxiliary equation are <u>real</u> and <u>distinct</u>

Section 31.8 repeated or complex roots

31.9 nonhomogeneous DE

<u>Ex:</u> Solve $D^3y = 4Dy$

$$y''' = 4y'$$

$$y''' - 4y' = 0$$

auxiliary equation $m^3 - 4m = 0$

$$m(m^2 - 4) = 0$$

$$m(m-2)(m+2)=0$$

$$m=0,\ 2,\ -2$$

$$\underbrace{y=C_1e^{m_1x}+C_2e^{m_2x}}_{\text{formula sheet}}+C_3e^{m_3x}$$

$$y=\cancel{C_1e^{0x}}\ C_1+C_2e^{2x}+C_3e^{-2x}\ \checkmark$$

$$y=C_1e^{2x}+C_2e^{-2x}+C_3\ \checkmark$$

etc.

Ex: Solve $y''-8y'=-5y$

$$y''-8y'+5y=0$$

$$m^2-8m+5=0$$

Quadratic Formula $\qquad \cancel{x}\ m=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

☆ Know this

Quadratic Formula

$$m = \frac{8 \pm \sqrt{(-8)^2 - 4\cdot1\cdot5}}{2}$$

$$= \frac{8 \pm \sqrt{44}}{2} \qquad \sqrt{4}\sqrt{11}$$

$$= \frac{8 \pm 2\sqrt{11}}{2}$$

$$= 4 \pm \sqrt{11}$$

Give exact values
(Use calculator to check)

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$y = C_1 e^{(4+\sqrt{11})x} + C_2 e^{(4-\sqrt{11})x}$$