Quiz tomorrow 31.2
Check www.camosun.ca for weather updates

31.6 Applications Part 1

Newton's Law of Cooling

Physical Principle: Rate at which an object cools/warms is proportional to the temperature difference between the object and the environment.

Ex: In a 25°C room
Coffee goes 90°C → 89°C faster than 67°C → 66°C
Cold water goes 4°C → 9°C " 12°C → 17°C

Build the DE:

T = object's temperature, t = time } variables

T_e = environment temp., k } constant

$$\frac{dT}{dt} \propto T - T_e$$

rate of change of temp. wrt time

$$\boxed{\frac{dT}{dt} = k(T - T_e)}$$ DE for heating/cooling

Formula will be provided

Ex: How long does it take a cup of coffee, initially 75°C, to cool to 40°C if it takes 6 minutes to cool to 60°C? Room temp. is 20°C.

Educated guess

$75°C \xrightarrow[6 \text{ mins}]{} 60°C \xrightarrow[>6 \text{ mins}]{} 45°C \xrightarrow[>0]{} 40°C$

So >12 mins

T: temp (°C)
t: time (mins)

DE:

$$\frac{dT}{dt} = k(T - T_e)$$

$$\frac{dT}{dt} = k(T - 20)$$

$$dT = k(T - 20)dt$$

$$\frac{dT}{T-20} = kdt \quad \text{variables are separated}$$

$$\int \frac{dT}{T-20} = \int kdt$$

$$\ln(T-20) = kt + C_1$$

$e^{LS} = e^{RS}$: $\quad e^{\ln(T-20)} = e^{kt+C_1}$

$$\boxed{e^{a+b} = e^a \cdot e^b}$$

$$T-20 = e^{kt}\cdot \cancel{e^{c_1}}\, C$$

$$\boxed{T = 20 + Ce^{kt}}$$

$T=75$, $t=0$:

$$75 = 20 + C(1)$$
$$C = 55$$

$$\boxed{T = 20 + 55e^{kt}}$$

$t=6$, $T=60$:

$$60 = 20 + 55e^{6k}$$
$$40 = 55e^{6k}$$
$$\frac{40}{55} = e^{6k}$$
$$\ln\left(\frac{40}{55}\right) = 6k$$
$$\frac{1}{6}\ln\left(\frac{40}{55}\right) = k$$

$$\boxed{T = 20 + 55e^{kt} \quad \text{k is known}}$$

$T=40$ (get t) :

$$40 = 20 + 55e^{kt}$$
$$20 = 55e^{kt}$$
$$\frac{20}{55} = e^{kt}$$
$$\ln\left(\frac{20}{55}\right) = kt$$

$$\frac{\ln\left(\frac{20}{55}\right)}{k} = t$$

$$t = \frac{\ln\left(\frac{20}{55}\right)}{\left(\frac{1}{6}\ln\left(\frac{40}{55}\right)\right)}$$

≈ 19 minutes

Physical Intuition:

As $t \to \infty$ what happens to T?

Temp $\to 20°C$

$T = 20 + 55e^{kt} \quad (k<0)$

As $t \to \infty$, $e^{kt} \to 0$ and $T \to 20$

$T = 20 + 55e^{kt}$