31.6 Applications Part I

Population Growth and Radioactive Decay

$$\boxed{\text{PHYSICAL PRINCIPLE: Rate of growth/decay is proportional to the quantity present}}$$

Let $N =$ quantity
$t =$ time

$$\frac{dN}{dt} \propto N$$

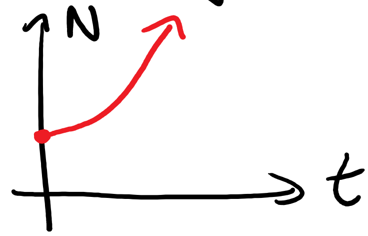
$$\boxed{\frac{dN}{dt} = kN \quad k: \text{Constant}}$$

DE for exponential growth/decay

☆ Know this

$k<0$
exponential decay

$k>0$
exponential growth



Ex: A radioactive substance has a half-life of 7.9 years. How long for 90% of the substance to decay?

for 90% of the substance to decay?

$\boxed{\text{half-life : time for 50\% of substance to decay}}$

$N =$ quantity

$N_0 =$ initial quantity

$t =$ time (years)

DE: $\frac{dN}{dt} = kN$ Separate variables

$$\frac{dN}{N\,dt} = k$$

$$\frac{dN}{N} = k\,dt \quad \text{separated}$$

$$\int \frac{dN}{N} = \int k\,dt$$

$$\ln N = kt + C_1$$

$e^{LS} = e^{RS}$:

$$e^{\ln N} = e^{kt + C_1}$$

$\boxed{e^{a+b} = e^a \cdot e^b}$

$$N = e^{kt} \cdot \cancel{e^{C_1}}_{C}$$

$$\boxed{N = Ce^{kt}}$$

Find C and k:

Sub $N = N_0$, $t = 0$: $N_0 = C\cancel{e^0}$

$C = N_0$

$$\boxed{N = N_0 e^{kt}}$$

Half-life

$N = 0.5 N_0$, $t = 7.9$: $0.5\cancel{N_0} = \cancel{N_0} e^{k(7.9)}$

$0.5 = e^{7.9k}$

$\ln 0.5 = \cancel{\ln e^{7.9k}}$ $7.9k$

$\frac{\ln 0.5}{7.9} = k$

$$\boxed{N = N_0 e^{\left(\frac{\ln 0.5}{7.9}\right)t}}$$

90% decayed

= 10% remaining

$N = 0.1 N_0$:

$0.1\cancel{N_0} = \cancel{N_0} e^{kt}$ (k is known)

$0.1 = e^{kt}$

$$\ln 0.1 = kt$$

$$\frac{\ln 0.1}{k} = t$$

$$t = \frac{\ln 0.1}{\left(\frac{\ln 0.5}{7.9}\right)}$$

$$\approx 26 \text{ years}$$

If we stated $N_0 = 100\,g$ @ beginning

Could sub $N = 50\,g$

$t = 7.9$ years

Ex: A population grows exponentially.

Population is 5.0 million in 2010

8.0 " 2012

Population in 2020 ?

$$\frac{dN}{dt} = kN$$

N: pop. (millions)

t: time (years after 2010)

⋮

$$\boxed{N = Ce^{kt}}$$ (see previous problem)

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$t = 0$
$N = 5$

$$5 = C\cancel{e^{0}}$$
$$C = 5$$

$$\boxed{N = 5e^{kt}}$$

$N = 8$
$t = 2$:

$$8 = 5e^{k(2)}$$

$$\frac{8}{5} = e^{2k}$$

$$\ln\left(\frac{8}{5}\right) = 2k$$

$$\frac{1}{2}\ln\left(\frac{8}{5}\right) = k$$

$$\boxed{N = 5e^{\left(\frac{1}{2}\ln\left(\frac{8}{5}\right)\right)t}}$$

Year 2020
$t = 10$:

$$N = 5e^{\left[\frac{1}{2}\ln\left(\frac{8}{5}\right)\cdot 10\right]}$$
$$\approx 52$$

$\boxed{\text{52 million}}$