Week 5 Thursday

February 7, 2019 7:39 AM

Quiz Tues 31.2

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31.4 Linear DE's Cont'd

Ex: Solve $\frac{dx}{dt} + 2tx = 3t$ with $x(0)=2$

1) Standard Form

$dy + P(x)\,y\,dx = Q(x)\,dx$

$y \to x$, $x \to t$

$dx + P(t)\,x\,dt = Q(t)\,dt$

Mult. by dt : $dx + 2tx\,dt = 3t\,dt$ ★

2) Find the integrating factor

~~$e^{\int P(x)dx}$~~ $e^{\int P(t)dt}$

$P(t) = 2t$

$\int P(t)dt = t^2$

$I.F. = e^{t^2}$

3) Multiply the standard form by the I.F.

$e^{t^2}dx + 2te^{t^2}x\,dt = 3te^{t^2}dt$ ★

Left side will be a differential

$d(e^{t^2} \cdot x) = 3te^{t^2}dt$

4) Integrate

$$\int d(e^{t^2}x) = \int 3te^{t^2}dt$$

Shortcut $\int d(e^{t^2}x) = \frac{3}{2}\int 2te^{t^2}dt$

$$e^{t^2}x = \frac{3}{2}e^{t^2} + C$$

Sub $u = t^2$

$du = 2tdt$

$\frac{du}{2} = tdt$

$\frac{3du}{2} = 3tdt$

Integral $= \frac{3}{2}\int e^u du$

$= \frac{3}{2}e^u + C$

$= \frac{3}{2}e^{t^2} + C$

5) Find C (if applicable)

$x(0) = 2 \rightarrow x = 2 \quad t = 0$

Sub $x = 2$, $t = 0$: $e^{t^2}x = \frac{3}{2}e^{t^2} + C$

$1(2) = \frac{3}{2}(1) + C$

$\frac{4}{2} = \frac{3}{2} + C$

$\frac{1}{2} = C$

$$e^{t^2}x = \frac{3}{2}e^{t^2} + \frac{1}{2}$$

Ex: Solve $x\,dy = y\,dx + x^5\,dx$

1) Standard Form

$dy + P(x)y\,dx = Q(x)\,dx$

$$dy = \frac{y}{x}dx + x^4dx$$

$$dy - \frac{y}{x}dx = x^4dx \quad \star$$

2) I.F. $= e^{\int P(x)dx}$

$P(x) = \frac{-1}{x}$

$\int P(x)dx = \int \frac{-1}{x}\,dx = -\ln x$

No +C
No abs. value

$e^{\int P(x)dx} = e^{-\ln x}$ SIMPLIFY

$= e^{\ln x^{-1}}$

$= x^{-1}$

3) Mult. the standard form by x^{-1}

$x^{-1}dy - \frac{y}{x}\cdot x^{-1}dx = x^{-1}x^4dx$

$\underbrace{x^{-1}dy - yx^{-2}dx} = x^3dx$

$d(?)$

$d(x^{-1}y) = x^3dx$

4) Integrate

$\int d(x^{-1}y) = \int x^3dx$

$x^{-1}y = \frac{x^4}{4} + C$

Ex: Identify separable or linear

a) $dx = 12x y^5 dy + 9x dy$

FACTOR

$dx = 3(4y^5 + 3)\, x\, dy$

$\frac{dx}{x} = 3(4y^5 + 3)\, dy$

Separable DE

Not linear (y^5)

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b) $x^2 dy = 9x^3 y dx + 8x^7 dx$

Linear ✓ $dy = 9x y dx + 8x^5 dx$

$dy - 9x y dx = 8x^5 dx$

$p(x) = -9x$ ...

Not separable

$x^2 dy = 9x^3 y dx + 8x^7 dx$

$x^2 dy = (9y + 8x^4) x^3 dx$