31.4 Linear DE's Cont'd

Recall $d(uv) = u\,dv + v\,du$

Ex: $d(x^4y) = x^4\,dy + y \cdot 4x^3\,dx$
$= x^4\,dy + 4x^3y\,dx$

$x^4\,dy + 4x^3y\,dx = d(?)$
$= d(x^4y)$ Shortcut

Ex: Write it as a differential:

$e^{-3x}\,dy - 3e^{-3x}y\,dx$

$= d(e^{-3x}y)$

Identify the dy term and replace the dy with y

To solve a linear DE:
use the integrating factor $e^{\int P(x)\,dx}$

Ex: Solve $\frac{dy}{y} - 4dx = \frac{e^{6x}}{y}dx$

1) Put it in standard form
$dy + P(x)\,y\,dx = Q(x)\,dx$

Mult by y: $dy - 4y\,dx = e^{6x}\,dx$ (★)

2) Calculate the integrating factor $e^{\int P(x)dx}$

$P(x) = -4$

$\int P(x)dx = -4x$ ← don't use $+C$

$e^{\int P(x)dx} = e^{-4x}$

3) Multiply the standard form by the integrating factor

Standard Form $dy - 4y\,dx = e^{6x}\,dx$

Mult. by e^{-4x}: $e^{-4x}\,dy - 4e^{-4x}y\,dx = e^{-4x} \cdot e^{6x}\,dx$

Left side is a differential
(The integrating factor guarantees this)

$d(e^{-4x}y) = e^{2x}\,dx$
Shortcut

4) Integrate

$\int dx = x + C \qquad \int du = u + C \qquad \int d(e^{-4x} y) = e^{-4x} y + C$

$$\int d(e^{-4x} y) = \int e^{2x} dx$$

$$e^{-4x} y = \frac{e^{2x}}{2} + C$$

Ex: Solve $\cos x \frac{dy}{dx} = 7 - y \sin x$

if $y\left(\frac{\pi}{3}\right) = 4$

1) Standard Form

$dy + P(x) y dx = Q(x) dx$

$$\cos x \, dy = 7 dx - y \sin x \, dx$$

$$dy = \frac{7}{\cos x} dx - \frac{y \sin x}{\cos x} dx$$

$$dy + \frac{\sin x}{\cos x} y dx = \frac{7}{\cos x} dx$$

$$dy + \tan x \, y dx = 7 \sec x \, dx \quad (\star)$$

2) $I.F. = e^{\int P(x) dx}$

$P(x) = \tan x$

$\int P(x) dx = \int \tan x dx = \ln \sec x$

No absolute value

No $+C$

$$e^{\int P(x) dx} = e^{\ln \sec x} = \sec x$$

3) Mult. the standard form ($\star$) by the I.F.

$\sec x\, dy + \sec x \tan x\, y\, dx = 7\sec^2 x\, dx$

Left side is a differential

$d(\sec x \cdot y) = 7\sec^2 x\, dx$

$d(y \sec x) = 7\sec^2 x\, dx$

4) Integrate

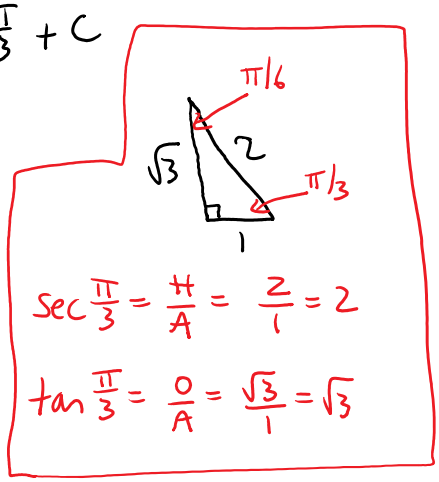
$\int d(y\sec x) = \int 7\sec^2 x\, dx$

$y\sec x = 7\tan x + C$

5) Find C (if applicable)

$y(\frac{\pi}{3}) = 4$

Sub $x = \frac{\pi}{3}$, $y = 4$: $4\sec\frac{\pi}{3} = 7\tan\frac{\pi}{3} + C$



$4(2) = 7\sqrt{3} + C$

$8 - 7\sqrt{3} = C$

$y\sec x = 7\tan x + 8 - 7\sqrt{3}$

Ex: Write it as a differential

$\ln x\, dy + \frac{1}{x} y\, dx$

$= d(\ln x \cdot y)$

Identify the dy term
Replace the dy with y

WHY?

$d(\ln x \cdot y) = \ln x\, dy + y \frac{1}{x} dx$ ✓