# Week 4 Friday

February 1, 2019 7:40 AM

## 31.2 Separation of Variables

We'll use $\frac{dy}{dx}$ rather than $y'$

Ex: Solve $dx = 12xy^5 dy$

Divide by $x$: $\frac{dx}{x} = 12y^5 dy$

"Variables are separated"

Integrate both sides:

$$\int \frac{dx}{x} = \int 12y^5 dy$$

$$\ln x + C_1 = 2y^6 + C_2$$

1) For simplicity, omit absolute value

$$\cancel{\ln|x| + C_1}$$

2) For simplicity:

$$\ln x = 2y^6 + \underbrace{\cancel{C_2 - C_1}}_{C}$$

Only need 1 constant (on the right side)

$$\ln x = 2y^6 + C \quad ✓$$

$$\ln x - 2y^6 = C \quad ✓$$

**Terminology:**

$\ln x = 2y^6 + C$ is an <u>implicit</u> solution

$y = \pm \sqrt[6]{\frac{\ln x - C}{2}}$ " <u>explicit</u> "

Give implicit solutions unless you're asked to find an explicit solution

**Ex:** Solve $3\frac{dy}{dx} = \frac{y(x+1)}{x}$

Mult. by dx: $3\,dy = \frac{y(x+1)}{x}\,dx$

Divide by y: $\frac{3\,dy}{y} = \frac{x+1}{x}\,dx$

$$\int \frac{3\,dy}{y} = \int \frac{x+1}{x}\,dx$$

$$\int \frac{3\,dy}{y} = \int \left(1+\frac{1}{x}\right)dx$$

$$3\ln y = x + \ln x + C$$

- no absolute value
- one constant

don't solve for y (unless asked)

**Recap:** $\displaystyle\int \frac{du}{u} = \ln|u| + C$ [ch 28]

**Shortcut:** $\displaystyle\int \frac{2x}{x^2+5}\,dx = \ln|x^2+5| + C$

$$\int \frac{3x}{x^2+5}\,dx = \int \frac{\left(\frac{3}{2}\right)2x}{x^2+5}\,dx = \frac{3}{2}\ln|x^2+5| + C$$

$3 = \,?\,(2)$

$$\int \frac{12x^2}{x^3+8}\,dx = \int \frac{(4)3x^2}{x^3+8}\,dx = 4\ln|x^3+8| + C$$

Log Rules:

$$n\ln a = \ln a^n$$
$$\ln a + \ln b = \ln(ab)$$
$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

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**Ex:** Solve $4xy\,dx + (x^2+1)\,dy = 0$

Divide by $y$: $\displaystyle 4x\,dx + \frac{(x^2+1)\,dy}{y} = 0$

Divide by $x^2+1$ :

$$\frac{4x\,dx}{x^2+1} + \frac{dy}{y} = 0$$

$$\int \frac{4x\,dx}{x^2+1} + \int \frac{dy}{y} = \int 0$$

$$\int \frac{2(2x)}{x^2+1}\,dx + \int \frac{dy}{y} = \int 0$$

$$2\ln(x^2+1) + \ln y = C$$

Follow-up Ex: Find an explicit solution (Solve for y)

$$\ln(x^2+1)^2 + \ln y = C$$

$$\ln\left[(x^2+1)^2 y\right] = C$$

$e^{LS} = e^{RS}$ :

$$e^{\ln\left[(x^2+1)^2 y\right]} = e^{C}$$

$$(x^2+1)^2 y = C_1$$

$$y = \frac{C_1}{(x^2+1)^2}$$