QUIZ

Find $\frac{\partial^2 z}{\partial x \partial y}$ for $z = y\sin 3x + x^4 y^2$

means $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$

$$\frac{\partial z}{\partial y} = \sin 3x + 2x^4 y$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(\sin 3x + 2x^4 y\right)$$

$2yx^4$

$$= 3\cos 3x + 8yx^3$$

Recall $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$ ✓

Notation $f(x,y) = \text{blah} \rightarrow \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

$z = \text{blah}$

$f = \text{blah}$

Ex: $f(x,y) = x^3 y^4$

$$\frac{\partial f}{\partial x} = 3x^2 y^4 \qquad \frac{\partial f}{\partial y} = 4x^3 y^3$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(3x^2 y^4\right)$$

$$= 12x^2 y^3$$

31.1 Cont'd

solution

Ex: a) Check that $y = c_1 x^{-1} + c_2 x^4$

solves $x^2y'' - 2xy' - 4y = 0$ ← DE

Solution
$$y = C_1 x^{-1} + C_2 x^4$$
$$y' = -C_1 x^{-2} + 4C_2 x^3$$
$$y'' = 2C_1 x^{-3} + 12C_2 x^2$$

DE:
$$LS = x^2 y'' - 2xy' - 4y$$
$$= x^2(2C_1 x^{-3} + 12C_2 x^2)$$
$$-2x(-C_1 x^{-2} + 4C_2 x^3)$$
$$-4(C_1 x^{-1} + C_2 x^4)$$
$$= 2C_1 x^{-1} + 12C_2 x^4$$
$$+ 2C_1 x^{-1} - 8C_2 x^4$$
$$- 4C_1 x^{-1} - 4C_2 x^4$$
$$= 0C_1 x^{-1} + 0C_2 x^4$$
$$= 0$$
$$= RS \quad \checkmark$$

b) Is y the general solution?

DEFINITION

General solution:

\# of unknown constants in solution = order of DE

$$y = C_1 x^{-1} + C_2 x^4$$
\# constants = 2 ✓

order of DE = 2

Yes

c) List some particular solutions

(# of unknown constants < order of DE)

General Solution $\quad y = C_1 x^{-1} + C_2 x^4$

Particular Solutions :

$$y = 0$$

$$y = 8x^4$$

$$y = -\sqrt{2}\, x^{-1}$$

$$y = \pi x^{-1} + e x^4$$

$$y = C_1 x^{-1} + 5x^4$$

More about Constants

$C_1 + C_2 x + C_3 x$ should be rewritten

$$= C_1 + (C_2 + C_3)x$$

$$= C_1 + C_4 x$$

$$\boxed{\text{Always collect like terms!}}$$

Ex: How many constants ?

$$A + 1 + C_1 - 5x^2 + C_2 x^2 + C_3 \ln x$$

$$= (A + 1 + C_1) + (-5 + C_2)x^2 + C_3 \ln x$$

$= C_4 + C_5 x^2 + C_3 \ln x$

$\boxed{3}$

<u>Ex</u>: The <u>general</u> solution of $y' - 3y = 6$

is $y = -2 + Ce^{3x}$

Find the <u>particular</u> solution if $y(0) = 7$

Rephrased $(x,y) = (0,7)$

General Solution $\boxed{y = -2 + Ce^{3x}}$

Sub $y=7$, $x=0$: $7 = -2 + C$

$9 = C$

$$\boxed{y = -2 + 9e^{3x}}$$