Week 4 Monday

January 28, 2019 7:30 AM

Quiz tomorrow 29.3

Math 193

1. CALCULUS ✓
2. DIFFERENTIAL EQUATIONS ←
3. STATS

31.1 Solutions of Differential Equations

Differential Equation (DE): An equation that contains at least 1 derivative

e.g. $y'' + 16y = 0$

$6y''' = y'$

We'll focus on:

1) Solving DE
2) Modelling real-world scenarios
   e.g. spring-mass system

The **order** of a DE is the order of the highest derivative in the DE

Ex: a) $6y''' = y'$ 3rd order DE

b) $8x^3y' = 4y$ 1st order DE

Notation

Recall y' can be written $\frac{dy}{dx}$

y'' " $\frac{d^2y}{dx^2}$

e.g. $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$

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means exactly the same thing as

$x^2y'' - 2xy' - 4y = 0$

Ex: a) Check that $y = 7e^{-x/2}$ ← Solution

is a solution to $2y' + y = 0$ ← DE

Solution $y = 7e^{-x/2}$

$y' = -\frac{7}{2}e^{-x/2}$

Check the DE

LS $= 2y' + y$ RS $= 0$

$= 2\left(-\frac{7}{2}e^{-x/2}\right) + 7e^{-x/2}$

$= -7e^{-x/2} + 7e^{-x/2}$

$= 0e^{-x/2}$

$= 0$

LS = RS ✓

b) Show that $y = x^2$ is <u>not</u> a solution

$y = x^2$

$y' = 2x$

Recall DE: $2y' + y = 0$

LS of DE $= 2y' + y$ RS of DE $= 0$

$= 2(2x) + x^2$

$= 4x + x^2$

$LS \neq RS$

Most functions are <u>not</u> solutions

Ex: Check that $y = C\sin 4x$ ← solution

solves $y'' + 16y = 0$ (C: constant)

DE

Solution $y = C\sin 4x$

$y' = 4C\cos 4x$

$y'' = -16C\sin 4x$

Check

$LS = y'' + 16y$ $\quad RS = 0$

$= -16C\sin 4x + 16(C\sin 4x)$

$= -16C\sin 4x + 16C\sin 4x$

$= 0C\sin 4x$

$= 0$

$LS = RS$ ✓

Ex: Check that $y = 5\tan 5x$ ← solution

is a solution of $y' = 25 + y^2$

DE

Solution $y = 5\tan 5x$

$y' = 25\sec^2 5x$

Check the DE : Start with the more complicated side

$RS = 25 + y^2$

$= 25 + (5\tan 5x)^2$

$= 25 + 25\tan^2 5x$

$= 25(1 + \tan^2 5x)$

$= 25\sec^2 5x$

$= y'$

$= LS$ ✓

$1 + \tan^2\theta = \sec^2\theta$

A solution that has # of unknown constants = order is called the general solution. Otherwise it's called a particular solution.

Ex: $y'' + 16y = 0$ has

general solution $y = C_1 \sin 4x + C_2 \cos 4x$

unknown constants

# of unknown constants = 2 = order of DE ✓

Some particular solutions:

$y = 0$

$y = 3\sin 4x$

$y = -\pi \cos 4x$

$y = 2\sin 4x + 8\cos 4x$

$y = C_1 \sin 4x + 2\cos 4x$