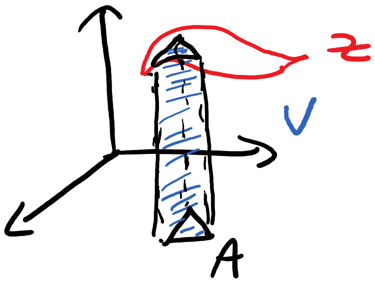
Week 3 Friday

January 25, 2019 7:45 AM

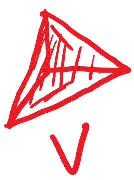
29.4 Double Integrals Cont'd

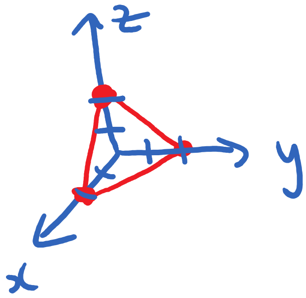


$V = \iint z \, dy \, dx$ (when using vertical slices for region A)

or $V = \iint z \, dx \, dy$ (" horizontal ")

Ex: Find the volume under $x+y+z=2$ in the first octant $(x, y, z \geq 0)$





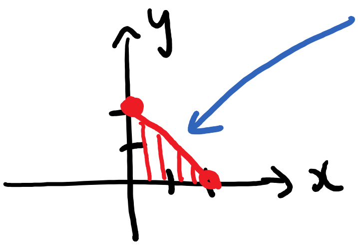
plane: $x+y+z=2$

$(0, 0, 2)$

$(0, 2, 0)$

$(2, 0, 0)$

1) Region A



$y = mx + b$
$y = -x + 2$

Vertical Slices

$0 \le y \le -x+2$
$0 \le x \le 2$

2) Find V

$$V = \iint z \, dy \, dx$$

$$= \underbrace{\int_0^2 \underbrace{\int_0^{-x+2} z \, dy} \, dx}$$

$x + y + z = 2$
$z = 2 - x - y$

$$= \int_0^2 \underbrace{\int_0^{-x+2} (2 - x - y) \, dy} \, dx$$

$$= \int_0^2 \left[2y - xy - \frac{y^2}{2} \right]_{y=0}^{y=-x+2} dx$$

$$= \int_0^2 \left[2(-x+2) - x(-x+2) - \tfrac{1}{2}(-x+2)^2 - 0 \right] dx$$

$$= \int_0^2 \left[-2x + 4 + x^2 - 2x - \tfrac{1}{2}(x^2 - 4x + 4) \right] dx$$

$$= \int_0^2 \left[-4x + 4 + x^2 - \tfrac{1}{2}x^2 + 2x - 2 \right] dx$$

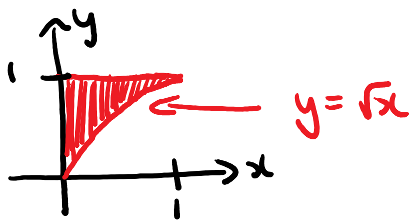
$$= \int_0^2 \left[-2x + 2 + \tfrac{1}{2}x^2 \right] dx$$

$$= \left[-x^2 + 2x + \frac{x^3}{6} \right]_0^2$$

$$= -4 + 4 + \frac{8}{6} - 0$$

$$= \frac{8}{6} \text{ or } \frac{4}{3}$$

Ex: Find volume under $z = e^{y^3}$ over the region:

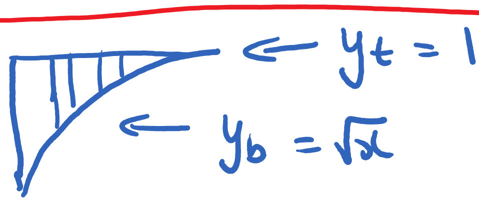


1) Region A

Vertical Slices

$\sqrt{x} \le y \le 1$

$0 \le x \le 1$



2) Find V

$$V = \iint z \, dy \, dx$$

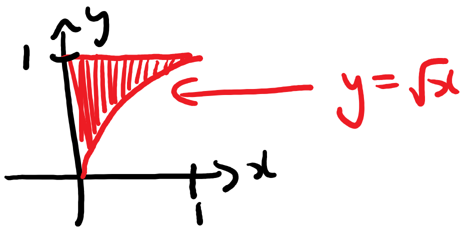
$$= \int_0^1 \int_{\sqrt{x}}^1 z \, dy \, dx$$

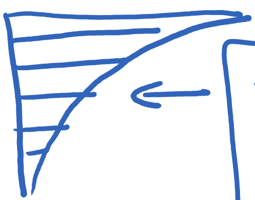
$$= \int_0^1 \int_{\sqrt{x}}^1 e^{y^3} \, dy \, dx$$

Can't integrate e^{y^3}!

— Try Horizontal Slices —

1) Region A





$x = 0$

$x_\ell = 0$

$y = \sqrt{x}$
$y^2 = x$
$x = y^2$

$x_r = y^2$

$0 \le x \le y^2$

$0 \le y \le 1$

2) Find V

$$V = \iint z \, dx \, dy$$

$$= \int_0^1 \int_0^{y^2} e^{y^3} \, dx \, dy$$

$$= \int_0^1 \left[e^{y^3} x \right]_{x=0}^{x=y^2} dy$$

$$= \int_0^1 e^{y^3} \cdot y^2 \, dy$$

Sub $u = y^3$

$du = 3y^2 dy$

$\frac{du}{3} = y^2 dy$

$y=0 \Rightarrow u=0$

$y=1 \Rightarrow u=1$

$$= \frac{1}{3} \int_0^1 e^u \, du$$

$$= \frac{1}{3} \left[e^u \right]_0^1$$

$$= \frac{1}{3}(e-1)$$