Week 3 Thursday

January 24, 2019 7:40 AM

Quiz Thes 29.3

Math Lab Mon-Thurs 11-4 TEC 142

Full Solutions to Sugg HW on D2L

29.4 Double Integrals

Computing volumes

Ex: $\int_1^2 \int_0^1 (x^2 + xy^3)\, dx\, dy$

Inside integral first

x is the variable (y is a #)

$$= \int_1^2 \left[\frac{x^3}{3} + \frac{x^2}{2} y^3\right]_{x=0}^{x=1} dy$$

$$= \int_1^2 \left[\frac{1}{3} + \frac{y^3}{2} - (0)\right] dy$$

$$= \int_1^2 \left[\frac{1}{3} + \frac{y^3}{2}\right] dy$$

$$= \left[\frac{y}{3} + \frac{y^4}{8}\right]_1^2$$

$$= \frac{2}{3} + \frac{16}{8} - \left(\frac{1}{3} + \frac{1}{8}\right)$$

$$= \frac{16}{24} + \frac{48}{24} - \left(\frac{8}{24} + \frac{3}{24}\right)$$

$$= \frac{53}{24}$$

Ex: $\int_0^2 \int_0^{x/2} xy^2 \, dy \, dx$

y is the variable

$$= \int_0^2 \left[\frac{xy^3}{3} \right]_{y=0}^{y=x/2} dx$$

$$= \int_0^2 \left[\frac{x}{3}\left(\frac{x}{2}\right)^3 - 0 \right] dx$$

$$= \int_0^2 \frac{x}{3}\left(\frac{x^3}{8}\right) dx$$

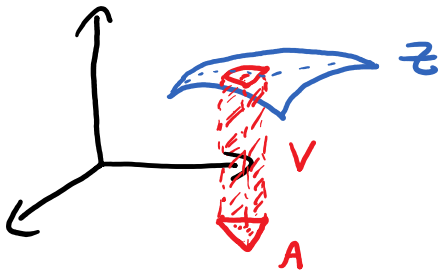
$$= \int_0^2 \frac{x^4}{24} \, dx$$

$$= \left[\frac{x^5}{120} \right]_0^2$$

$$= \frac{2^5}{120} - 0$$

$$= \frac{32}{120} \text{ or } \frac{4}{15}$$

Volume under a surface z over a region A

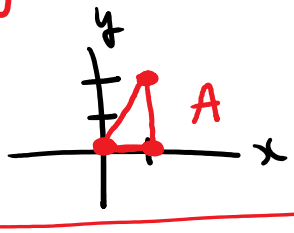


$$V = \iint z \, dy \, dx$$

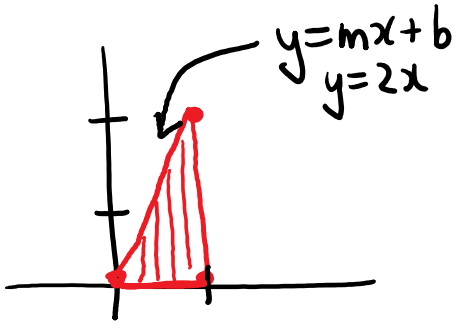
[or $V = \iint z \, dx \, dy$]

Ex: Find volume bounded by $z = xy$ over the region bounded by $(x,y) = (0,0)$, $(1,0)$ and $(1,2)$.

1) Region A



Vertical Slices



$0 \le y \le 2x$ — Dependent Variable

$0 \le x \le 1$ — Independent

2) Find V

$$V = \iint z \, dy \, dx$$

Dependent

$$= \iint xy \, dy \, dx$$

$$= \int_0^1 \int_0^{2x} xy \, dy \, dx$$

$$= \int_0^1 \left[\frac{xy^2}{2} \right]_{y=0}^{y=2x} dx$$

$$= \int_0^1 \left[\frac{x(2x)^2}{2} - 0 \right] dx$$

$$= \int_0^1 2x^3 \, dx$$

$$= \left[\frac{2x^4}{4} \right]_0^1$$

$$= \frac{2}{4} - 0$$

$$= \frac{1}{2}$$