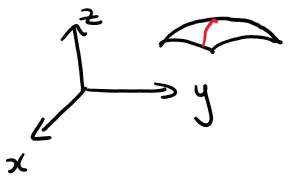
Quiz Tues 29.3

29.3 Cont'd

Partial Derivatives



slice parallel to x-axis

slope of the tangent line is $\frac{\delta z}{\delta x}$

"partial derivative of z with respect to x"

Notation: $\frac{\partial z}{\partial x}$ or $\frac{\delta z}{\delta x}$ or z_x

Ex: $f = x^3 + 4xy + 7y$ $\quad (4y)_x$

$\frac{\partial f}{\partial x}$: x is the variable (y is constant)

$$\frac{\partial f}{\partial x} = 3x^2 + 4y + 0$$
$$= 3x^2 + 4y$$

$\frac{\partial f}{\partial y}$: y is the variable (x is constant)

$$\frac{\partial f}{\partial y} = 0 + 4x + 7$$
$$= 4x + 7$$

Ex: $f = \cos xy$

$$\frac{\partial f}{\partial y} = -\sin xy \, (x)$$
$$= -x \sin xy$$

$$\frac{\partial f}{\partial x} = -\sin xy \, (y)$$

$$\frac{\partial f}{\partial x} = -\sin xy \,(y)$$
$$= -y\sin xy$$

Ex: $z = \dfrac{e^{6x+y}}{x^2+7}$

Find $\left.\dfrac{\partial z}{\partial x}\right|_{(0,0,\frac{1}{7})}$

Recall Quotient Rule $\left(\dfrac{u}{v}\right)' = \dfrac{vu' - v'u}{v^2}$

$\dfrac{\partial z}{\partial x}$: x is the variable (y is a #)

$$\frac{\partial z}{\partial x} = \frac{(x^2+7)(6e^{6x+y}) - (2x)(e^{6x+y})}{(x^2+7)^2}$$

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=0\\y=0}} = \frac{7(6)-0}{7^2}$$
$$= \frac{42}{49} \text{ or } \frac{6}{7}$$

Second-order Partial Derivatives

Notation

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial^2 z}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

Ex: $z = x^2\cos 4y$ Find :

a) $\dfrac{\partial^2 z}{\partial x^2}$

$\frac{\partial^2 z}{\partial x^2}$

$$\frac{\partial z}{\partial x} = 2x\cos 4y$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(2x\cos 4y\right)$$

$$= 2\cos 4y$$

b) $$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial x} = 2x\cos 4y$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\left(2x\cos 4y\right)$$

$$= 2x \cdot -4\sin 4y$$

$$= -8x\sin 4y$$

$$\boxed{z = x^2\cos 4y}$$

c) $$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial z}{\partial y} = x^2 \cdot -4\sin 4y$$

$$= -4x^2\sin 4y$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(-4x^2\sin 4y\right)$$

$$= -8x\sin 4y$$

Notice $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$

Always true in Math 193

[In general, $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$ when $\frac{\partial^2 z}{\partial y \partial x}$ and $\frac{\partial^2 z}{\partial x \partial y}$ are continuous]

d) $$\frac{\partial^2 z}{\dots} = \frac{\partial}{\dots}\left(\frac{\partial z}{\dots}\right)$$

d) $\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$

$\frac{\partial z}{\partial y} = -4x^2 \sin 4y$

$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial y}\left(-4x^2 \sin 4y\right)$

$= -4x^2 \cdot 4\cos 4y$

$= -16x^2 \cos 4y$

ASIDE

$$\boxed{\begin{aligned} \frac{d}{dy} \sin 4y &= 4\cos 4y \\ \frac{d}{dy} -\sin 4y &= -4\cos 4y \\ \frac{d}{dy} -2\sin 4y &= -8\cos 4y \end{aligned}}$$