

- Omit Section 28.9 # 11, 19
- Try Mixed Integration Problems [www.leahhoward.com](http://www.leahhoward.com)

Quiz tomorrow 28.7

Test Thurs 31<sup>st</sup>  
28.1-4, 28.6-7, 28.9, 29.3-4

- 1) Do Sugg HW
- 2) Practice Problems [www.leahhoward.com](http://www.leahhoward.com)

BRING CALCULATOR

## 28.9 Partial Fractions Cont'd

Shortcut  $\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$

Ex:  $\int \frac{5x+1}{x^2+5x} dx$   
 $= \int \frac{5x+1}{x(x+5)} dx$

$$\frac{5x+1}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5} \quad A, B: \text{Constants}$$

"partial fraction expansion"

Multiply by  $x(x+5)$ :

$$5x+1 = \frac{A}{\cancel{x}} \cancel{x}(x+5) + \frac{B}{\cancel{x+5}} \cancel{x}(x+5)$$

$$5x+1 = A(x+5) + Bx \quad \text{"Master Equation"}$$

$$Sx+1 = A(x+5) + Bx \quad \text{"Master Equation"}$$

Sub appropriate values into Master Equation

$$x=0 : \quad 1 = A(5)$$

$$A = \frac{1}{5}$$

$$x=-5 : \quad -24 = B(-5)$$

$$B = \frac{24}{5}$$

Conclude:

$$\begin{aligned} \frac{Sx+1}{x(x+5)} &= \frac{A}{x} + \frac{B}{x+5} \\ &= \frac{1}{5} \cdot \frac{1}{x} + \frac{24}{5} \cdot \frac{1}{x+5} \end{aligned}$$

$$\begin{aligned} \text{Integral} &= \int \left( \frac{1}{5} \cdot \frac{1}{x} + \frac{24}{5} \cdot \frac{1}{x+5} \right) dx \\ &= \frac{1}{5} \ln|x| + \frac{24}{5} \ln|x+5| + C \end{aligned}$$

$$\underline{\text{Ex:}} \quad \int \frac{x+1}{x(x-1)(2x+1)} dx$$

$$\frac{x+1}{x(x-1)(2x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1} \quad A, B, C : \#$$

$$x+1 = \frac{A}{x} \cancel{x(x-1)(2x+1)} + \frac{B}{x-1} \cancel{x(x-1)(2x+1)} + \frac{C}{2x+1} \cancel{x(x-1)(2x+1)}$$

$$x+1 = A(x-1)(2x+1) + Bx(2x+1) + Cx(x-1)$$

Sub in values that make some terms = 0

$$x=0: \quad 1 = A(-1)(1) \\ A = -1$$

$$x-1=0 \\ x=1: \quad 2 = B(1)(3) \\ B = \frac{2}{3}$$

$$2x+1=0 \\ 2x=-1 \\ x = -\frac{1}{2}: \quad -\frac{1}{2} + 1 = 0A + 0B + C\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) \\ \frac{1}{2} = \frac{3}{4}C \\ \frac{4}{3} \cdot \frac{1}{2} = C \\ C = \frac{2}{3}$$

$$\frac{x+1}{x(x-1)(2x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$$

$$= \frac{-1}{x} + \frac{2}{3} \cdot \frac{1}{x-1} + \frac{2}{3} \cdot \frac{1}{2x+1}$$

$$\text{Integral} = \int \left( \frac{-1}{x} + \frac{2}{3} \cdot \frac{1}{x-1} + \frac{2}{3} \cdot \frac{1}{2x+1} \right) dx$$

$$= -\ln|x| + \frac{2}{3} \ln|x-1| + \frac{2}{3} \cdot \frac{1}{2} \ln|2x+1| + C \\ \frac{1}{3}$$