Week 2 Friday

January 18, 2019 7:34 AM

28.7 Integration By Parts Cont'd

- Tabular Method
- $\int u\,dv = uv - \int v\,du$

Ex: $\int \ln x \, dx$

D	I
1	$\ln x$
	?

	D	I	
u	$\ln x$	1	dv
du	$\frac{1}{x}$ or x^{-1}	x	v
	$\vdots$	$\vdots$	

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x \, dx = x\ln x - \int x \cdot x^{-1} \, dx$$

$$= x\ln x - \int 1 \, dx$$

$$= x\ln x - x + C$$

Ex: $\int \tan^{-1} x \, dx$

D	I
1	$\tan^{-1} x$
	?

	D	I	
u	$\tan^{-1} x$	1	dv
du	$\frac{1}{1+x^2}$	x	v
	$\vdots$	$\vdots$	

$$\int u\,dv = uv - \int v\,du$$

$$\int \tan^{-1}x \, dx = x\tan^{-1}x - \int \frac{x}{1+x^2} \, dx$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int \frac{x}{1+x^2} \, dx$$

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u| + C_1$$

$$= \frac{1}{2}\ln|1+x^2| + C_1$$

$$\int \tan^{-1}x \, dx = x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + C$$

Ex: $\int e^x \cos x \, dx$

	D	I	
u	$\cos x$	e^x	dv
du	$-\sin x$	e^x	v
	$\vdots$	$\vdots$	

(or other way around)

$$\int u\,dv = uv - \int v\,du$$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx \quad (A)$$

Do Integration by Parts again

	D	I	
u	$\sin x$	e^x	dv
du	$\cos x$	e^x	v

$$\int u\,dv = uv - \int v\,du$$

$$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$$

Plug $\int e^x \sin x\,dx$ into Ⓐ

Ⓐ $$\int e^x \cos x\,dx = e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x + C_1$$

$\div 2$:

$$\int e^x \cos x\,dx = \frac{1}{2}\left[e^x \cos x + e^x \sin x\right] + C_2$$

Tips for Choosing u and dv:

Integral	u	dv
$\int x^n e^x\,dx$	x^n	e^x
$\int x^n \cos x\,dx$	x^n	$\cos x$
$\int x^n \sqrt{1+x}\,dx$	x^n	$\sqrt{1+x}$
[illegible]	[illegible]	x^n

} Tabular Method

$\int x^n \ln x \, dx$	$\ln x$	x^n
$\int x^n \tan^{-1} x \, dx$	$\tan^{-1} x$	x^n

} Formula

28.9 Partial Fractions

Ex: $\int \frac{1}{2x+3} dx$ (28.2)

$u = 2x+3$
$du = 2dx$
$\frac{du}{2} = dx$

$= \frac{1}{2} \int \frac{du}{u}$

$= \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|2x+3| + C$

Shortcut $\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$

Ex: $\int \frac{dx}{x-2} = \ln|x-2| + C$

$\int \frac{dx}{7x+4} = \frac{1}{7} \ln|7x+4| + C$