Week 2 Thursday

January 17, 2019 7:53 AM

Quiz Tues Section 28.7

28.7 Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

Ex: $\int x\cos x\,dx$

$u = x$	$dv = \cos x\,dx$
$du = dx$	$v = \int \cos x\,dx$
	$= \sin x$ ~~+C~~

$$\int u\,dv = uv - \int v\,du$$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + C$$

Choose u so that du is simpler than u

Shortcut: "Tabular Method"

	D	I
+	x	$\cos x$
−	1	$\sin x$
	0	$-\cos x$

Answer: $x\sin x - 1(-\cos x) + C$

$= x\sin x + \cos x + C$

Ex: $\int r^3 e^{2r}\,dr$

	D	I
⊕	r^3	e^{2r}
⊖	$3r^2$	$e^{2r}/2$
⊕	$6r$	$e^{2r}/4$
⊖	6	$e^{2r}/8$
	0	$e^{2r}/16$

$$\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$$

Answer: $\frac{r^3 e^{2r}}{2} - \frac{3r^2 e^{2r}}{4} + \frac{6re^{2r}}{8} - \frac{6e^{2r}}{16} + C$

$= \left(\frac{r^3}{2} - \frac{3r^2}{4} + \frac{3r}{4} - \frac{3}{8}\right)e^{2r} + C$

Ex: $\int 14x^3 \sin x \cos x\,dx$ 7.2

$2\sin x\cos x = \sin 2x$

$= \int 7x^3 \sin 2x\,dx$

	D	I
⊕	$7x^3$	$\sin 2x$

⊕	$7x^3$	$\sin 2x$
⊖	$21x^2$	$-\frac{\cos 2x}{2}$
⊕	$42x$	$-\frac{\sin 2x}{4}$
⊖	42	$\frac{\cos 2x}{8}$
	0	$\frac{\sin 2x}{16}$

Answer: $-\frac{7x^3\cos 2x}{2} + \frac{21x^2\sin 2x}{4} + \frac{42x\cos 2x}{8} - \frac{42\sin 2x}{16} + C$

Ex: $\int 3x\sqrt{1+x}\,dx$

	D	I
⊕	$3x$	$(1+x)^{1/2}$
⊖	3	$\frac{2}{3}(1+x)^{3/2}$
	0	$\frac{4}{15}(1+x)^{5/2}$

[Sub $u=1+x$]

$\frac{2}{3}\cdot\frac{2}{5}$

Answer: $2x(1+x)^{3/2} - \frac{4}{5}(1+x)^{5/2} + C$

Ex: $\int x \ln x \, dx$

D	I
x	$\ln x$
	?

	D	I	
(u)	$\ln x$	x	(dv)
(du)	$\frac{1}{x} = x^{-1}$	$\frac{x^2}{2}$	(v)
	$-x^{-2}$	$\vdots$	
	$\vdots$		

If both columns continue forever, use $\int u\,dv = uv - \int v\,du$

$$\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int x^{-1} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2} \, dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$