Week 2 Tuesday

January 15, 2019 8:25 AM

Quiz $\int_0^{\pi/2} \frac{\cos x\,dx}{1+\sin x}$

$u = 1+\sin x$

$du = \cos x\,dx$

$x=0 \to u=1$

$x=\frac{\pi}{2} \to u=2$

$= \int_1^2 \frac{du}{u}$

$= [\ln|u|]_1^2$

$= \ln|2| - \ln|1|$

$= \ln 2 - \ln 1$

$= \ln 2$

28.4 Cont'd

Ex: $\int \sqrt{\tan^2 7x + 1}\,dx$

$1+\tan^2\theta = \sec^2\theta$

$= \int \sqrt{\sec^2 7x}\,dx$

$= \int \sec 7x\,dx$

$= \frac{1}{} \ln|\sec 7x + \tan 7x| + C$

$$= \frac{1}{7}\ln|\sec 7x + \tan 7x| + C$$

Ex: $\int \frac{dx}{1-\sin x}$ TRICKY

Multiply top and bottom by "conjugate"

$$= \int \frac{(1+\sin x)}{(1+\sin x)(1-\sin x)} \cdot dx$$

$$= \int \frac{(1+\sin x)\,dx}{1-\sin^2 x}$$

$\sin^2 x + \cos^2 x = 1$

$$= \int \frac{(1+\sin x)\,dx}{\cos^2 x}$$

$$= \int\left[\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right] dx$$

$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$$= \int (\sec^2 x + \sec x \tan x)\,dx$$

$$= \tan x + \sec x + C$$

28.6 Inverse Trig Forms

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$a > 0$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$$

On formula sheet, in terms of x

Ex: Evaluate

a) $\int \frac{dx}{\sqrt{36-7x^2}}$

$= \int \frac{dx}{\sqrt{6^2-(\sqrt{7}x)^2}}$

$u = \sqrt{7}x$

$du = \sqrt{7}\,dx$

$\frac{du}{\sqrt{7}} = dx$

$= \frac{1}{\sqrt{7}}\int \frac{du}{\sqrt{6^2-u^2}}$

$= \frac{1}{\sqrt{7}}\sin^{-1}\left(\frac{u}{6}\right)+C$

$= \frac{1}{\sqrt{7}}\sin^{-1}\left(\frac{\sqrt{7}x}{6}\right)+C$

b) $\int_{0.1}^{0.2} \frac{e^x\,dx}{\sqrt{9-e^{2x}}}$

$e^{2x} = (e^x)^2$

$= \int_{}^{0.2} e^x\,dx$

$$= \int_{0.1}^{0.2} \frac{e^x\,dx}{\sqrt{3^2-(e^x)^2}}$$

$u = e^x$
$du = e^x dx$
$x = 0.1 \to u = e^{0.1}$
$x = 0.2 \to u = e^{0.2}$

$$= \int_{e^{0.1}}^{e^{0.2}} \frac{du}{\sqrt{3^2-u^2}}$$

$$= \left[\sin^{-1}\left(\frac{u}{3}\right)\right]_{e^{0.1}}^{e^{0.2}}$$

$$= \sin^{-1}\left(\frac{e^{0.2}}{3}\right) - \sin^{-1}\left(\frac{e^{0.1}}{3}\right)$$

Radian Mode

$$\approx 0.04$$

Ex: Evaluate $\int \frac{\cos x\,dx}{9+\sin^2 x}$

$$= \int \frac{\cos x\,dx}{3^2+(\sin x)^2}$$

$u = \sin x$
$du = \cos x\,dx$

$$= \int \frac{du}{3^2+u^2}$$

$$= \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C \qquad a=3$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{\sin x}{3}\right)+C$$