

Quiz tomorrow 28.2

28.4 Basic Trig Integration

Recall $\frac{d}{dx} [\sin x] = \cos x$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

Know these
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8 other formulas (all on formula sheet):

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln |\sec x| + C$$

$$\int \cot x \, dx = -\ln |\csc x| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$\int \csc x \, dx = -\ln |\csc x + \cot x| + C$$

Ex: $\int \cos 4x dx$

$$\begin{aligned} u &= 4x \\ du &= 4dx \\ \frac{du}{4} &= dx \end{aligned}$$

$$= \frac{1}{4} \int \cos u du$$

$$= \frac{1}{4} \sin u + C$$

$$= \frac{1}{4} \sin 4x + C$$

Shortcut: When angle = kx

$$\int \sin 3x = -\frac{\cos 3x}{3} + C$$

$$\int \sec 7x \tan 7x dx = \frac{\sec 7x}{7} + C$$

$$\int \csc 8x dx = -\frac{1}{8} \ln |\csc 8x + \cot 8x| + C$$

Ex: $\int x \sec x^2 \tan x^2 dx$

$$\begin{aligned} u &= x^2 \\ du &= 2x dx \\ \frac{du}{2} &= x dx \end{aligned}$$

$$= \frac{1}{2} \int \sec u \tan u du$$

$$= \frac{1}{2} \sec u + C$$

$$= \frac{1}{2} \sec x^2 + C$$

Ex: a) $\int \frac{\tan x}{\cos x} dx$

REWRITE

$$= \int \sec x \tan x dx$$

$$= \sec x + C$$

b) $\int \frac{1}{\sin^2 x} dx$

$$= \int \csc^2 x dx$$

$$= -\cot x + C$$

$$\cot x = \frac{1}{\tan x} \quad \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x}$$

Ex: $\int_0^{\sqrt{\pi}} 3x \tan x^2 dx$

$$\begin{aligned} u &= x^2 \\ du &= 2x dx \\ \frac{du}{2} &= x dx \end{aligned}$$

$$= \frac{3}{2} \int_0^{\pi/4} \tan u \, du$$

$$= \frac{3}{2} [\ln |\sec u|]_0^{\pi/4}$$

$$= \frac{3}{2} [\ln |\sqrt{2}| - \ln |1|]$$

$$= \frac{3}{2} [\ln \sqrt{2} - \ln 1]$$

$$= \frac{3}{2} \ln \sqrt{2}$$

Ex:

$$\int_0^{\pi/12} \frac{dx}{\cos^2 2x}$$

$$= \int_0^{\pi/12} \sec^2 2x \, dx$$

$$= \left[\frac{\tan 2x}{2} \right]_0^{\pi/12}$$

$$\frac{du}{2} = x dx$$

$$\frac{3du}{2} = 3x dx$$

$$x=0 \rightarrow u=0$$

$$x = \frac{\sqrt{\pi}}{2} \rightarrow u = \frac{\pi}{4}$$

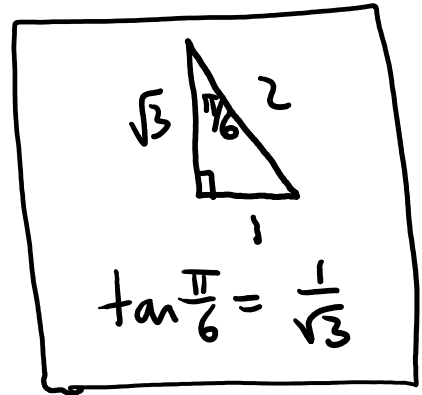


$$\cos 0 = 1 \quad \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sec 0 = 1 \quad \sec \frac{\pi}{4} = \sqrt{2}$$

Shortcut

$$= \frac{1}{2} (\tan \frac{\pi}{6} - \tan 0)$$



$$= \frac{1}{2} \left(\frac{1}{\sqrt{3}} - 0 \right)$$

$$= \frac{1}{2\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}}{6}$$

Ex: $\int \csc 2x (\sin 2x + \cot 2x) dx$

$$= \int (1 + \csc 2x \cot 2x) dx$$

$$= x - \frac{\csc 2x}{2} + C$$