Week 13 Monday

April 1, 2019 7:35 AM

Integral of the Day

$$\int \frac{2\,dx}{(\tan^{-1}x)(1+x^2)}$$

$$\boxed{\begin{aligned} u &= \tan^{-1}x \\ du &= \frac{1}{1+x^2}\,dx \end{aligned}}$$

$$= \int \frac{2\,du}{u}$$

$$= 2\ln|u| + C$$

$$= 2\ln|\tan^{-1}x| + C$$

- Quiz tomorrow Section 5
- Test Thursday
  Stats 1-7
  Sugg HW and Practice Problems (www.leahhoward.com)

Section 8. Central Limit Theorem Cont'd

RECAP

- Use $z = \frac{x-\mu}{\sigma}$ for x = single measurement

  Valid when X is normal

- Use $z = \frac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$ for $\bar{x}$ = sample mean
  = average of n measurements

  Valid when $n \geq 30$

Ex 1. A large class has a test average of 72 with a SD of 8. Take a random sample of n tests. Find the probability that the average of the n tests is more than 75 if:

a) $n = 30$

b) $n = 80$

a) pop. mean $\mu = 72$ $\quad \sigma = 8 \quad n = 30$

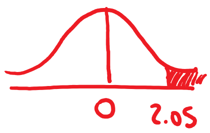
$P(\bar{x} > 75)$

sample mean

$n \geq 30$ ✓

$$\boxed{\begin{aligned} z &= \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})} \\ &= \frac{75 - 72}{(8/\sqrt{30})} \\ &\approx 2.05 \end{aligned}}$$

$= P(z > 2.05)$

$=$ 

$= 0.5 - 0.4798$

$= 0.0202$

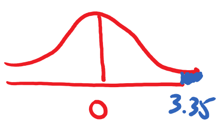
b) $n = 80$

$P(\bar{x} > 75)$

$n \geq 30$ ✓

$$\boxed{\begin{aligned} z &= \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})} \\ &= \frac{75 - 72}{(8/\sqrt{80})} \\ &\approx 3.35 \end{aligned}}$$

$= P(z > 3.35)$

$=$ 

$= 0.5 - 0.4996$

$= 0.0004$

Note: As n gets larger, $\bar{x}$-values stay closer to μ

Ex 2. Checked baggage has a mean mass of 21 kg with a SD of 4 kg. If 40 bags are randomly selected, find the probability that their average mass is:
a) between 20 and 23 kg
b) less than 20 kg or more than 22kg

a) $\mu = 21 \quad \sigma = 4 \quad n = 40$

$P(20 \le \bar{x} \le 23)$

$n \geq 30$ ✓

$z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$

$z_1 = \dfrac{20-21}{(4/\sqrt{40})} \approx -1.58$

$z_2 = \dfrac{23-21}{(4/\sqrt{40})} \approx 3.16$

$= P(-1.58 \le z \le 3.16)$

$=$

$= 0.4429 + 0.4992$

$= 0.9421$

b) P(less than 20 kg or more than 22 kg)

$= P(\bar{x} < 20 \text{ or } \bar{x} > 22)$

$n \geq 30$ ✓

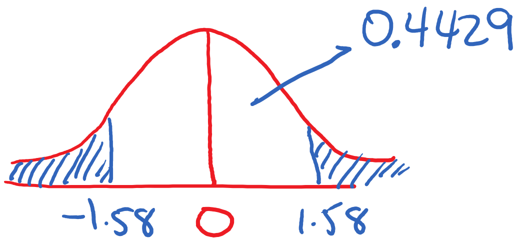
$$z = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})}$$

$z_1 \approx -1.58$

$z_2 \approx 1.58$

$= P(z < -1.58 \text{ or } z > 1.58)$

$=$



$= 1 - 2(0.4429)$ or $2[0.5 - 0.4429]$

$= 0.1142$

Ex 3. Checked baggage has a mean mass of 21 kg with a SD of 4 kg. Find the probability that the total mass of 50 random bags is greater than 1130 kg.

$\mu = 21 \quad \sigma = 4 \quad n = 50$

$P(\text{total mass} > 1130 \text{ kg})$

total mass = 1130 kg

$$\frac{\text{total mass}}{50} = \frac{1130 \text{ kg}}{50}$$

average mass = 22.6

$$\bar{x} = 22.6$$

$$= P(\bar{x} > 22.6)$$

$n \geq 30$ ✓

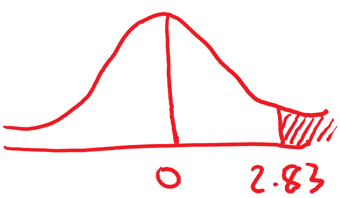
$$z = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})}$$

$$= \frac{22.6 - 21}{(4/\sqrt{50})}$$

$$\approx 2.83$$

$$= P(z > 2.83)$$

=



$$= 0.5 - 0.4977$$

$$= 0.0023$$