Integral of the Day

$$\int \frac{x\,dx}{4+x^4}$$

$$= \int \frac{x\,dx}{2^2+(x^2)^2}$$

$u = x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$$= \frac{1}{2}\int \frac{du}{2^2+u^2}$$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$\int \frac{du}{2^2+u^2} = \frac{1}{2}\tan^{-1}\frac{u}{2} + C$

$$= \frac{1}{2}\left(\frac{1}{2}\tan^{-1}\frac{u}{2}\right) + C$$

$$= \frac{1}{4}\tan^{-1}\frac{x^2}{2} + C$$

6. Cont'd

For a continuous random variable X with p.d.f. $f(x)$,
the mean or expected value of X is

$$\mu \text{ or } E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

The variance of X is

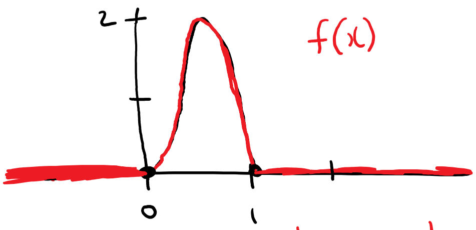
$$\sigma^2 = E(X^2) - \mu^2 \quad \text{where } E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

Ex 4. The proportion X of a city's roads needing repair in any given year has p.d.f.

$$f(x) = \begin{cases} 12x^2 - 12x^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Graph $f(x)$ using Wolfram Alpha
b) Find the expected proportion of roads needing repairs this year
c) Find the SD of X

a)



X= proportion of roads needing repair

b) Find μ or $E(x)$

$$\mu \text{ or } E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\boxed{f(x) = \begin{cases} 12x^2 - 12x^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}}$$

$$= \cancel{\int_{-\infty}^{0} 0} + \int_0^1 \underbrace{(12x^3 - 12x^4)}_{x f(x)}dx + \cancel{\int_1^{\infty} 0}$$

$$= \left[\frac{12x^4}{4} - \frac{12x^5}{5}\right]_0^1$$

$$= \left(\frac{12}{4} - \frac{12}{5}\right) - 0$$

$$= 0.6$$

c) Variance σ^2 first
Then σ

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$f(x) = \begin{cases} 0, & x < 0 \\ 12x^2 - 12x^3, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

$$= \cancel{\int_{-\infty}^{0} x^2 \cdot 0} + \int_0^1 (12x^4 - 12x^5)\,dx + \cancel{\int_1^{\infty} 0}$$

$$\cancel{\int x \cdot 0} + \int_0^1 (12x^4 - 12x^5)\,dx + \cancel{\int_1 0}$$

$$= \left[\frac{12x^5}{5} - \frac{12x^6}{6}\right]_0^1$$

$$= \left(\frac{12}{5} - 2\right) - 0$$

$$= 0.4$$

$$\sigma^2 = E(x^2) - \mu^2$$

$$= 0.4 - (0.6)^2$$

$$= 0.04 \quad \text{variance}$$

SD $\sigma = \sqrt{0.04}$

$$= 0.2$$

Omit Examples 5-8 from Section 6

Omit Sugg HW Section 6 #4

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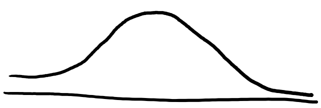
7. The Normal Distribution

Used for mound-shaped data like lengths and masses



We standard the x-values using the z-score $\boxed{z = \frac{x-\mu}{\sigma}}$

then use z-table (avoids integration)

Ex 1. The volume in bottles of ginger ale is normally distributed with a mean of 2.01 L and a SD of 0.13 L. Find the probability that a bottle has a volume:
a) between 1.77 and 2.29 L
b) between 1.59 and 1.73 L
c) less than 1.81 L

a) X = volume (L)

$P(1.77 \leq X \leq 2.29)$

X is normal → use z

$z = \dfrac{x-\mu}{\sigma}$ $\quad \mu = 2.01 \quad \sigma = 0.13$

$z = \dfrac{1.77 - 2.01}{0.13} \approx -1.85$

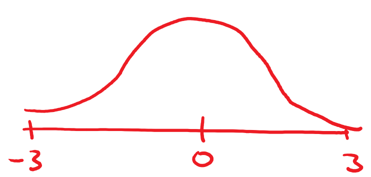
SD's away from the mean

$z = \dfrac{2.29 - 2.01}{0.13} \approx 2.15$

$= P(-1.85 \leq z \leq 2.15)$

z-curve is centred at 0

±3 are extreme values



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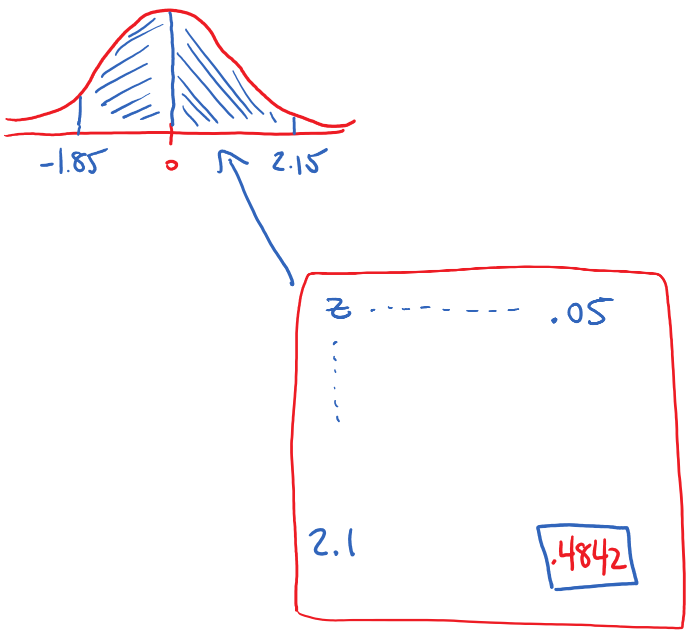

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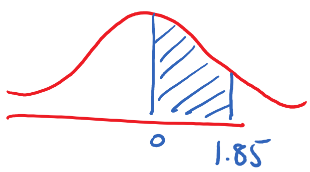

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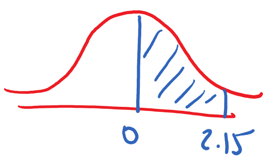

Use symmetry

$= 0.4678 + 0.4842$

$= 0.952$