Week 10 Thursday

March 14, 2019 8:03 AM

Integral of the Day

$$\int \frac{\tan x}{\cos x}\,dx$$

$$\cos x = \frac{1}{\sec x}$$

$$\frac{1}{\cos x} = \sec x$$

$$= \int \sec x \tan x\,dx$$

$$= \sec x + C$$

Other Identities

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

Integration Formulas

$$\int \sin u\,du = -\cos u + C$$

$$\int \cos u\,du = \sin u + C$$

Section 3 Probability

Probability: measure of likelihood

$0 \leq$ probability ≤ 1

(0%) — event is impossible

(100%) — event is guaranteed to happen

Sample space of an experiment: set of possible outcomes

Event: subset of the sample space

> Notation: S for sample space
> A, B, C etc. for events

Ex 1. Flip a fair coin three times. What is the probability of getting one or two heads?

Sample space $S = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$

Event $A = \{THH, HTH, HHT, TTH, THT, HTT\}$

desired outcomes: 1 or 2 heads

$$P(A) = \frac{n(A) \leftarrow \#\text{ of outcomes in } A}{n(S) \leftarrow \#\text{ of outcomes in } S}$$

"probability of A" $= \frac{6}{8}$ or $\frac{3}{4}$ or 0.75 or 75%

Notation: $P(A)$ is the probability of getting an outcome in A.

FACT

When outcomes are equally likely,

$$P(A) = \frac{n(A)}{n(S)}$$

Ex 2. Randomly select an integer between 1 and 40 (inclusive). Find the probability of getting a multiple of 5 or 7.

$S = \{1, 2, \cdots, 39, 40\}$

$A = \{\underbrace{5, 10, 15, 20, 25, 30, 35, 40}_{\text{mult. of 5}}, \underbrace{7, 14, 21, 28}_{\text{mult. of 7}}\}$

Don't count 35 twice!

$P(A) = \frac{12}{40}$

Ex 3. Roll a pair of fair 6-sided dice. Find the probability of getting a sum of at most 5.

Die #1 \ Die #2	1	2	3	4	5	6
1	★	★	★	★		
2	★	★	★			
3	★	★				
4	★					
5						
6						

$n(S) = 6 \times 6 = 36$

Note: One way to roll double 1's

Two ways a 1 and a 2

A: sum ≤ 5 (★)

$n(A) = 10$

$P(A) = \frac{10}{36}$

Ex 4. Four study groups have the following numbers of students: 4, 6, 7, 9. Pick two of the groups at random. Find the probability that they have at least 15 students in total.

$S = \{\{4,6\}, \{4,7\}, \{4,9\}, \{6,7\}, \{6,9\}, \{7,9\}\}$

$\{6,4\} = \{4,6\}$ $n(S) = 6$

$\{6,4\} = \{4,6\}$ $\quad n(S) = 6$

$A = \{ \{6,9\}, \{7,9\} \}$

sum to ≥ 15

$P(A) = \frac{2}{6}$ or 33%