Week 1 Friday

January 11, 2019 7:28 AM

Quiz Tues 28.2
- Closed book
- Bring your calculator

28.3 Cont'd

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1) \qquad [28.1]$$

$$\int \frac{du}{u} = \ln|u| + C \qquad [28.2]$$

$$\int e^u du = e^u + C \qquad [28.3]$$

Shortcut $\int e^{kx} dx = \frac{e^{kx}}{k} + C$

Ex: Evaluate $\int \frac{7e^{6x}}{e^{2x}} dx$

Simplify $= \int 7e^{4x} dx$

$= \frac{7}{4} e^{4x} + C$

Ex: $\int_{-2}^{-1} \frac{dx}{x^2 e^{\frac{1}{x}}}$

$$= \int_{-2}^{-1} \frac{e^{-\frac{1}{x}}\,dx}{x^2}$$

$u = \frac{-1}{x}$ or $-x^{-1}$

$du = x^{-2}dx$ or $\frac{dx}{x^2}$

when $x=-2$, $u=\frac{1}{2}$

$x=-1$, $u=1$

$$= \int_{\frac{1}{2}}^{1} e^u\,du$$

$$= \left[e^u\right]_{\frac{1}{2}}^{1}$$

$$= e - e^{\frac{1}{2}}$$

Alternative (without u-values)

$$\int_{-2}^{-1} \frac{e^{-1/x}\,dx}{x^2}$$

$u = -\frac{1}{x}$

$du = \frac{dx}{x^2}$

$$= \int_{x=-2}^{x=-1} e^u\,du$$

$$= \left[e^u\right]_{x=-2}^{x=-1}$$

$$= \left[e^{-\frac{1}{x}}\right]_{-2}^{-1}$$ ⇐ messier algebra

$$= e - e^{1/2}$$

Ex: $\int_0^{\pi/6} \cos 3\theta \, e^{\sin 3\theta} \, d\theta$

$u = \sin 3\theta$

$du = 3\cos 3\theta \, d\theta$

$\frac{du}{3} = \cos 3\theta \, d\theta$

$\theta = 0 \rightarrow u = \sin 0 = 0$

$\theta = \frac{\pi}{6} \rightarrow u = \sin \frac{\pi}{2} = 1$

$$= \frac{1}{3} \int_0^1 e^u \, du$$

$$= \frac{1}{3} \left[e^u \right]_0^1$$

$$= \frac{1}{3} (e - 1)$$

Alternative (without u-values)

$$\int_0^{\pi/6} \cos 3\theta \, e^{\sin 3\theta} d\theta$$

$u = \sin 3\theta$

$du = 3\cos 3\theta \, d\theta$

$\frac{du}{3} = \cos 3\theta \, d\theta$

:) no u-values

$$= \frac{1}{3} \int_{\theta=0}^{\theta=\frac{\pi}{6}} e^u \, du$$

$$= \frac{1}{3} \left[e^u \right]_{\theta=0}^{\theta=\frac{\pi}{6}}$$

$$= \frac{1}{3} \left[e^{\sin 3\theta} \right]_{\theta=0}^{\theta=\pi/6}$$

$$= \frac{1}{3} \left[e^{\sin \frac{\pi}{2}} - e^{\sin 0} \right]$$

:(messier algebra

$$= \frac{1}{3} (e - 1)$$

Ex: $\int_0^{\pi/2} \sin 2x \, e^{\sin^2 x} \, dx$

Hint: $\sin 2x = 2 \sin x \cos x$ (formula sheet)

$u = \sin^2 x$ or $[\sin x]^2$

$du = 2 \sin x \cos x \, dx$

$x = 0 \rightarrow u = [\sin 0]^2 = 0$

$x=0 \to u=[\sin 0]^2=0$

$x=\frac{\pi}{2} \to u=[\sin\frac{\pi}{2}]^2=1$

$$= \int_0^{\pi/2} 2\sin x\cos x\, e^{\sin^2 x}\, dx$$

$$= \int_0^1 e^u\, du$$

$$= [e^u]_0^1$$

$$= e-1$$

Ex: $\int \sqrt{e^{4y}+e^{6y}}\, dy$ (Section 28.1)

TRICKY

$$= \int \sqrt{e^{4y}(1+e^{2y})}\, dy$$

Simplify

$$= \int \sqrt{e^{4y}}\sqrt{1+e^{2y}}\, dy$$

$\sqrt{x^4}=x^2$

$\sqrt{e^{4y}}=e^{2y}$

$$= \int e^{2y}\sqrt{1+e^{2y}}\, dy$$

$u=1+e^{2y}$

$du=2e^{2y}\,dy$

$\frac{du}{2}=e^{2y}\,dy$

$$= \frac{1}{2}\int \sqrt{u}\, du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)+C$$

$$= \frac{1}{3}(1+e^{2y})^{3/2}+C$$