28.2 Cont'd

The Logarithmic Form

$$\int \frac{du}{u} = \ln|u| + C$$

Recap

$$n\ln a = \ln a^n$$

$$\ln a + \ln b = \ln(ab)$$

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

Ex: Evaluate $\int \tan x\,dx$

$$= \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$= -\int \frac{du}{u}$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

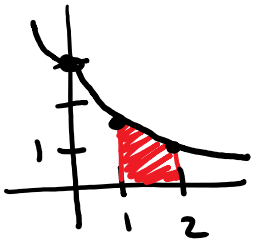
OR $\ln|\cos x|^{-1} + C$

$n\ln a = \ln a^n$

$$= \ln\frac{1}{|\cos x|} + C$$

$$= \ln|\sec x| + C$$

Ex: Find the area under $y = \frac{3}{x+1}$ from $x=1$ to $x=2$.



x	y
0	3
1	3/2
2	1

$$A = \int_1^2 \frac{3}{x+1}\,dx$$

$u = x+1$
$du = dx$
when $x=1$, $u=2$
$x=2$, $u=3$

$$= 3\int_2^3 \frac{du}{u}$$

$$= 3\left[\ln|u|\right]_2^3$$

$$= 3\left[\ln|3| - \ln|2|\right]$$

$$= 3\left[\ln 3 - \ln 2\right] \checkmark$$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$$= 3\ln\left(\frac{3}{2}\right) \checkmark$$

28.3 The Exponential Form

$$\int e^u\,du = e^u + C$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

Contrast with 28.1 $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$

28.2 $\int \frac{du}{u} = \ln|u| + C$

Ex: Evaluate $\int x e^{2x^2} dx$

$u = 2x^2$

$du = 4x\,dx$

$\frac{du}{4} = x\,dx$

$= \frac{1}{4}\int e^u du$

$= \frac{1}{4} e^u + C$

$= \frac{1}{4} e^{2x^2} + C$

Ex: $\int e^{4x} dx$

$u = 4x$

$du = 4\,dx$

$\frac{du}{4} = dx$

$= \frac{1}{4}\int e^u du$

$= \frac{1}{4} e^u + C$

$= \frac{1}{4} e^{4x} + C$

Shortcut $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

k: Constant $\neq 0$

Ex: $\int \frac{dx}{\sqrt{e^x}}$

$\frac{1}{\sqrt{a}} = a^{-1/2}$

Rewrite!

$= \int (e^x)^{-1/2}\,dx$

$= \int e^{-\frac{1}{2}x}\,dx$

Sub
$u = -\frac{1}{2}x$
$du = -\frac{1}{2}dx$
$-2du = dx$

Integral $= -2\int e^u\,du$

$= -2e^u + C$

$= -2e^{-\frac{1}{2}x} + C$

Shortcut
$\frac{1}{(-\frac{1}{2})}e^{-\frac{1}{2}x} + C$

$= -2e^{-\frac{1}{2}x} + C$

Ex: $\int e^{6x}(e^{2x} + e^{-6x})\,dx$ (expand)

$= \int (e^{8x} + e^0)\,dx$

$= \int (e^{8x} + 1)\,dx$

$= \frac{e^{8x}}{8} + x + C$