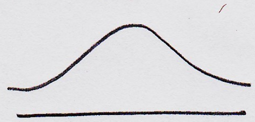
7. The Normal Distribution

Used for mound-shaped continuous data like lengths and masses.



We standardize x-values using the z-score $z = \frac{x-\mu}{\sigma}$. Then use z-table.

<u>Ex</u>: The volume in bottles of ginger ale is normally distributed with a mean of 2.01L and a SD of 0.13L. Find the probability that a bottle has a volume:

a) between 1.77 and 2.29L

X = volume (L) $\quad \mu = 2.01 \quad \sigma = 0.13$

Want $P(1.77 \le X \le 2.29)$

X is normal $\rightarrow$ use $z = \frac{x-\mu}{\sigma}$

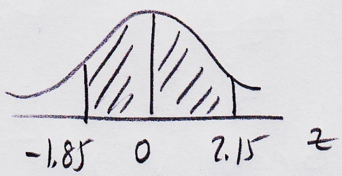
$z_1 = \frac{1.77-2.01}{0.13} \approx -1.85 \qquad z_2 = \frac{2.29-2.01}{0.13} \approx 2.15$

$P(1.77 \le X \le 2.29)$

$= P(-1.85 \le z \le 2.15)$

$\boxed{\begin{array}{l} \text{z-curve is centred at 0} \\ \pm 3 \text{ are extreme values} \\ \text{Probability = area under curve} \end{array}}$

$=$



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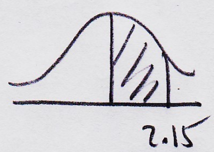

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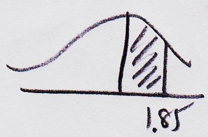

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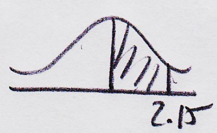


symmetry

z	…	.05
⋮		
1.8		$\boxed{.4678}$

$= 0{,}4678 + 0{,}4842$

$= 0.952$

b) between 1.59 and 1.73 L

$$z = \frac{x - \mu}{\sigma}$$

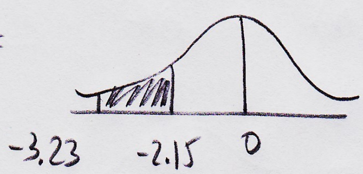
$$z_1 = \frac{1.59 - 2.01}{0.13} \simeq -3.23$$

$$z_2 = \frac{1.73 - 2.01}{0.13} \simeq -2.15$$

$$P(1.59 \le X \le 1.73)$$

$$= P(-3.23 \le z \le -2.15)$$

=



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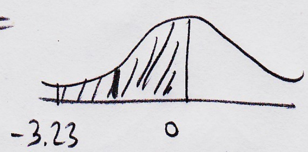

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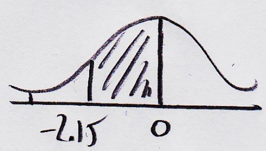


Use symmetry
Look up $z = 3.23, 2.15$

$$= 0.4994 - 0.4842$$

$$= 0.0152$$

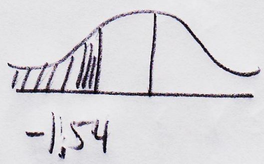
c) less than 1.81L

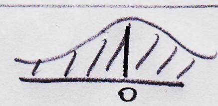
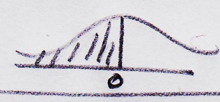
$$z = \frac{1.81 - 2.01}{0.13} \approx -1.54$$

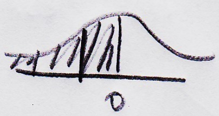
$P(X < 1.81)$

$= P(z < -1.54)$

$=$



Total area = 1	
Half-area = 0.5	

$=$ 

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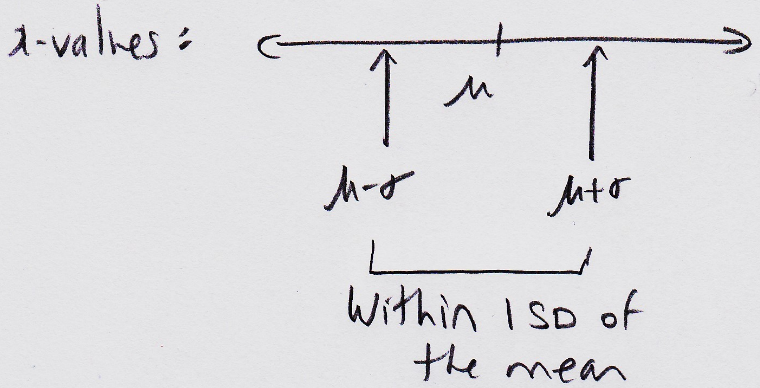
$= 0.5 - 0.4382$

$= 0.0618$

<u>Ex</u>: Find the proportion of x-values that are within 1 SD of the mean for a normal distribution.



Want $P(\mu-\sigma \le X \le \mu+\sigma)$

$$\boxed{\begin{array}{ll} \text{NORMAL} & \\ z_1 = \dfrac{(\mu-\sigma)-\mu}{\sigma} & z_2 = \dfrac{(\mu+\sigma)-\mu}{\sigma} \\ \quad = -1 & \quad = 1 \end{array}}$$

$= P(-1 \le z \le 1)$

$=$

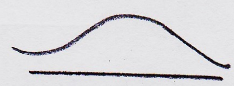
$= 0.3413 + 0.3413$

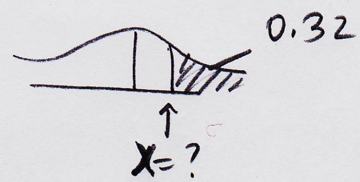
$= 0.6826$

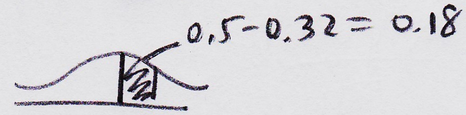
Ex: The mass of a brand of a chocolate bar is normal with a mean of 85g and a SD of 1.5g. Find the mass that separates the largest 32% of chocolate bar masses from the others.

X = mass (g) $\quad \mu = 85 \quad \sigma = 1.5$

NORMAL







$$\boxed{\begin{array}{l}\text{Reverse look-up area} = 0.18 \\ \text{Choose closest z-score}\end{array}}$$

$z = 0.47$

$$z = \frac{x - \mu}{\sigma}$$

$$0.47 = \frac{x - 85}{1.5}$$

$$1.5(0.47) = x - 85$$

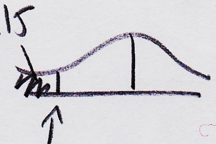
$$x = 85 + 1.5(0.47) \approx 86\text{ g}$$

Ex: The lengths of drill bits are normal with a mean of 4.2cm and a SD of 1.1cm. Find the length that separates the smallest 15% of drill bit lengths from the others.

X = length (cm) $\quad \mu = 4.2 \quad \sigma = 1.1$

NORMAL

0.15



$X = ?$

$0.5 - 0.15 = 0.35$

Reverse look-up area $= 0.35$

$z = 1.04$

Use $z = -1.04$ by symmetry

$$z = \frac{x - \mu}{\sigma}$$

$$-1.04 = \frac{x - 4.2}{1.1}$$

$$-1.04(1.1) = x - 4.2$$

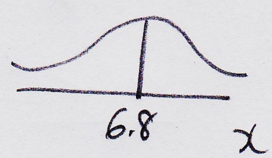
$$x = 4.2 - 1.04(1.1) \approx 3.1 \text{ cm}$$

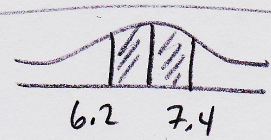
Ex: The time it takes to inspect a ball bearing is normal with a mean of 6.8 seconds. Find the SD of the inspection times if 26.62% of inspection times are between 6.2 and 7.4 seconds.

X = time (s) $\mu = 6.8$

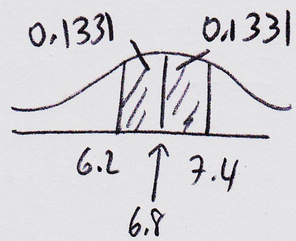
NORMAL

x-values are centred at the mean because $x = \mu$ corresponds to $z = 0$





symmetric about 6.8



total area = 0.2662
half-area = 0.1331

$$\boxed{\begin{array}{c}\text{Reverse look-up area} = 0.1331 \\ z = 0.34\end{array}}$$

Use $x = 7.4$ $z = 0.34$ $\mu = 6.8$

$$z = \frac{x - \mu}{\sigma}$$

$$0.34 = \frac{7.4 - 6.8}{\sigma}$$

$$0.34 = \frac{0.6}{\sigma}$$

$$\sigma = \frac{0.6}{0.34} \approx 1.8 \text{ seconds}$$