4. Discrete Random Variables

Discrete random variable: function that assigns a number to each outcome of an experiment

Notation: X for a discrete random variable
x for a specific value of X

The **probability distribution** of X is a table:

x	$P(x)$
x_1	p_1
x_2	p_2
$\vdots$	$\vdots$

Ex: X = # heads observed in 3 coin tosses
Find the probability distribution of X

x	Outcomes	# Outcomes	$P(x)$
0	TTT	1	$\frac{1}{8} = 0.125$
1	HTT, THT, TTH	3	0.375
2	HHT, HTH, THH	3	0.375
3	HHH	1	0.125
		$\boxed{\text{total} = 8}$	

Probability Distribution of X:

x	$P(x)$
0	0.125
1	0.375
2	0.375
3	0.125

The **mean** or **expected value** of X is $\mu = E(x) = \sum x P(x)$

The **variance** of X is

$\sigma^2 = E(X^2) - \mu^2$ where $E(X^2) = \sum x^2 P(x)$

The **SD** of X is $\sigma = \sqrt{\sigma^2}$

Ex:

x	$P(x)$
-5	0.15
-2	0.2
1	0.4
6	0.25

Find :

a) $P(-2.5 \le X \le 2.5)$

$= P(X=-2) + P(X=1)$

$= 0.2 + 0.4$

$= 0.6$

b) the mean (or expected value) of X

$\mu = E(X) = -5(0.15) + (-2)(0.2)$
$+ 1(0.4) + 6(0.25)$
$= 0.75$

c) the variance of X

$E(X^2) = \sum x^2 p(x)$
$= 25(0.15) + 4(0.2) + 1(0.4) + 36(0.25)$
$= 13.95$

$\sigma^2 = E(X^2) - \mu^2$
$= 13.95 - 0.75^2$
$= 13.3875$

d) the standard deviation of X

$$\sigma = \sqrt{\sigma^2}$$
$$= \sqrt{13.3875}$$
$$\approx 3.66$$

e) P(a value of X lies with 1 SD of the mean)

$$= P(\mu - \sigma \le X \le \mu + \sigma)$$
$$= P(-2.91 \le X \le 4.41)$$
$$= P(X = -2) + P(X = 1)$$
$$= 0.2 + 0.4$$
$$= 0.6$$

<u>Ex</u>: Project 1 has a 35% chance of earning \$0

Project 1	35%	\$0
	50%	\$300,000
	15%	\$800,000
Project 2	60%	\$0
	40%	\$1,000,000

a) Find the probability distributions of the earnings for each project.

Let X = earnings for Project 1 (\$)

x	$P(x)$
0	0.35
300,000	0.5
800,000	0.15

Let Y = earnings for Project 2 (\$)

y	$P(y)$
0	0.6
1,000,000	0.4

b) Find the expected earnings for each project

$$\mu_X = E(X) = 0(0.35) + 300{,}000(0.5) + 800{,}000(0.15)$$
$$= \$270{,}000$$

$$\mu_Y = E(Y) = 0(0.6) + 1{,}000{,}000(0.4)$$
$$= \$400{,}000$$

c) Find the SD of earnings for each project

$$E(X^2) = 0^2(0.35) + (300{,}000)^2(0.5) + (800{,}000)^2(0.15)$$

$$= 1.41 \times 10^{11}$$

$$\sigma_X^2 = E(X^2) - \mu_X^2$$
$$= 1.41 \times 10^{11} - (270{,}000)^2$$
$$= 6.81 \times 10^{10} \quad (\$^2)$$

$$\sigma_X = \sqrt{\sigma_X^2}$$
$$\approx \$261{,}000$$

$$E(Y^2) = 0^2(0.6) + (1{,}000{,}000)^2(0.4)$$
$$= 4 \times 10^{11}$$

$$\sigma_Y^2 = E(Y^2) - \mu_Y^2$$
$$= 4 \times 10^{11} - (400{,}000)^2$$
$$= 2.4 \times 10^{11} \quad (\$^2)$$

$$\sigma_Y = \sqrt{\sigma_Y^2}$$
$$\approx \$490{,}000$$

d) which project has higher expected earnings

Project 2 because $E(Y) > E(X)$

e) In terms of earnings, which project is riskier?

Project 2 because $\sigma_Y > \sigma_X$

Conclusion: Project 2 is riskier with higher average reward

Ex: You want to insure a \$2,000 tablet against theft for 1 year by paying a premium m. The probability of theft is 4.7%

a) Find the probability distribution of the insurance Company's gain

Let X = gain (\$)

= premium − payout

	x	$P(x)$
no theft	m	$0.953 \leftarrow \boxed{1-0.047}$
theft	$m-2000$	0.047

b) Find the premium if the insurer expects to gain \$40

$$E(x) = 40$$

$$m(0.953) + (m-2000)(0.047) = 40$$

$$0.953m + 0.047m - 94 = 40$$

$$m = 134$$

$$\boxed{\text{Premium} = \$134}$$