# 3. Probability

**Probability:** measure of likelihood

$0 \leq$ probability $\leq 1$

$0$: impossible; $1$: certain

**Sample space** of an experiment: set of possible outcomes

**Event:** subset of sample space

Notation: $S$ for sample space
$A, B$ etc. for events

**Ex:** Flip a fair coin 3 times
Probability of getting 1 or 2 heads?

$S = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$

$A = \{1 \text{ or } 2 \text{ heads}\}$

$= \{THH, HTH, HHT, TTH, THT, HTT\}$

$$P(A) = \frac{n(A)}{n(S)} \quad \begin{array}{l} \leftarrow \text{\# outcomes in } A \\ \leftarrow \text{\# outcomes in } S \end{array}$$

$$= \frac{6}{8}$$

Interpretation: $P(A)$ is the probability of getting an outcome in $A$

> When outcomes are equally likely
> $$P(A) = \frac{n(A)}{n(S)}$$

Ex: Pick a random # between 1 and 40 (inclusive). P(getting a multiple of 5 or 7)?

$S = \{1, 2, \ldots, 40\}$

$A = \{5, 10, 15, 20, 25, 30, 35, 40, 7, 14, 21, 28\}$

Note: Don't list "35" twice

$$P(A) = \frac{n(A)}{n(S)} = \frac{12}{40}$$

Ex: Roll two fair dice. P(rolls sum to at most 5)?

| Die #1 \ Die #2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | ★ | ★ | ★ | ★ | | |
| 2 | ★ | ★ | ★ | | | |
| 3 | ★ | ★ | | | | |
| 4 | ★ | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

Note: 1 way to get double 1's
2 ways to get a 1 and a 2

$n(S) = 6 \times 6 = 36$

$n(A) = 10$ ★

$P(A) = \frac{10}{36}$

Ex: 4 study groups have the following numbers of students: 4, 6, 7, 9.
Pick 2 groups randomly. P(they have at least 15 students in total)?

$S = \{\{4,6\}, \{4,7\}, \{4,9\}, \{6,7\}, \{6,9\}, \{7,9\}\}$

$A = \{\{6,9\}, \{7,9\}\}$

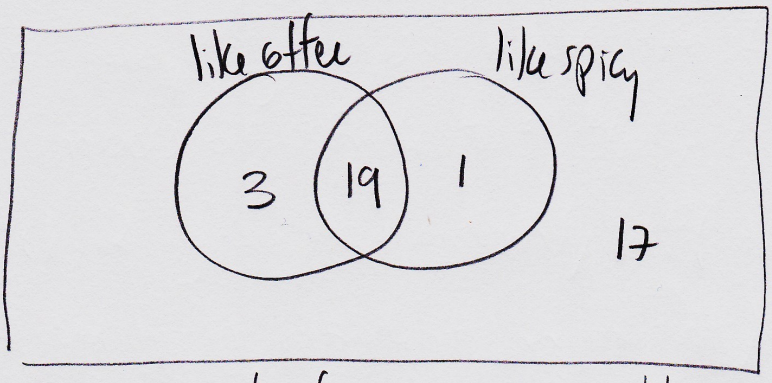
$P(A) = \frac{2}{6}$

When outcomes are not equally likely we use a table or Venn diagram.

Ex: 2 ways to present same data
Consider 40 Math 193 Students

| | Like Coffee | Dislike Coffee |
|---|---|---|
| Like Spicy Food | 19 | 1 |
| Dislike Spicy | 3 | 17 |

"table"




one circle for each variable
"Venn diagram"

Ex: Using data above, find the probability that a student:

a) likes coffee

$n(S) = 40$

$n(A) = 19 + 3 = 22$

$P(A) = \frac{22}{40}$

b) likes coffee and doesn't like spicy

$\frac{3}{40}$

b) likes coffee or dislikes spicy

$n(\text{likes coffee or dislikes spicy}) = 19 + 3 + 17 = 39$

$\text{probability} = \frac{39}{40}$

Ex: In a class of 45 students, 26 have jobs and 17 have cars. Of those who don't have a car, 10 have jobs. Find the probability that a student has:

a) a car or a job

→ Table or Venn diagram

| | Car | No Car |
|---|---|---|
| Job | | 10 |
| No Job | | |

| | C | no C |
|---|---|---|
| J | 16 | 10 |
| no J | | |

| | C | no C |
|---|---|---|
| J | 16 | 10 |
| no J | 1 | |

| | C | no C | |
|---|---|---|---|
| J | 16 | 10 | |
| no J | 1 | 18 | ← 45 − 16 − 10 − 1 |

$$P(\text{car or job}) = \frac{16+1+10}{45} = \frac{27}{45}$$

b) a car but not a job

$$P(\text{car and no job}) = \frac{1}{45}$$

Ex: Machine I breaks down 4% of the time

Machine II " 7% "

Both machines " 2% "

P( Machine II breaks down and I doesn't)?

→ Table or Venn diagram

| I \ II | ok | down |
|---|---|---|
| ok | | |
| down | | 0.02 |

| I \ II | ok | down |
|---|---|---|
| ok | | 0.05 |
| down | 0.02 | 0.02 |

| I \ II | ok | down |
|---|---|---|
| ok | 0.91 | 0.05 |
| down | 0.02 | 0.02 |

Probabilities sum to 100%

1 − 0.05 − 0.02 − 0.02

P(II down and I ok) = 0.05

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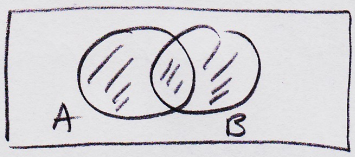
## 3 Probability Rules

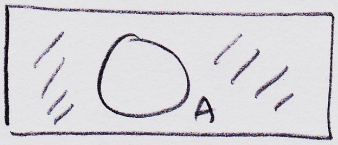
1) $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$




2) $n(\text{not } A) = n(S) - n(A)$

$P(\text{not } A) = 1 - P(A)$




3) Multiply # of outcomes when performing a sequence of activities

Ex: # of possible outcomes when we roll a die 5 times?

6 outcomes for each roll

$n(S) = 6 \times 6 \times 6 \times 6 \times 6 = 6^5$

Ex: A password consists of 7 digits, each chosen from 0-9. Find:

a) total # of passwords

$n(S) = 10 \times 10 \times \dots \times 10 = 10^7$

b) # of passwords that end with 3

$\_ \; \_ \; \_ \; \_ \; \_ \; \_ \; \boxed{3}$

$10 \times 10 \times \dots \times 10 = 10^6$

c) # of passwords that don't end with 3

$n(S) - n(\text{end with } 3)$

$= 10^7 - 10^6$

$= 9{,}000{,}000$

d) P(password starts with 4)

$\boxed{4} \; \_ \; \_ \; \_ \; \_ \; \_ \; \_$

fixed

$n(\text{start with } 4) = 10^6$

$P(\text{start with } 4) = \frac{10^6}{10^7} = 0.1$

e) P(doesn't start with 4)

$= 1 - P(\text{starts with } 4)$

$= 0.9$

f) P(password contains at least one 4)

n(contain at least one 4)

$= n(S) - n(\text{contain no 4's})$

_ _ _ _ _ _ _ _

must be chosen from 0-3, 5-9

$= 10^7 - 9^7$

$P(\text{Contains at least one 4}) = \frac{10^7 - 9^7}{10^7}$

$\simeq 0.52$

g) P(starts with 29 or ends with 1)

n(start 29 or end 1)

$= n(\text{start } 29) + n(\text{end } 1) - n(\text{start } 29 \text{ and end } 1)$

29 _ _ _ _ _

29 _ _ _ _ 1

$= 10^5 + 10^6 - 10^4$

$$\text{Probability} = \frac{10^5 + 10^6 - 10^4}{10^7}$$

$$= 0.109$$