29.3 Intro to Surfaces

Three Methods for Graphing Surfaces

Ex: Graph $x+2y+3z=6$ ← surface in 3D

Let $x=0$ and $y=0$ $\quad 3z=6$

$z=2$

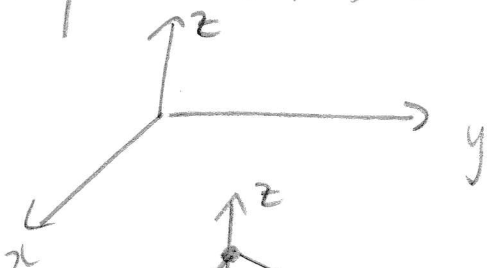
$(0,0,2)$

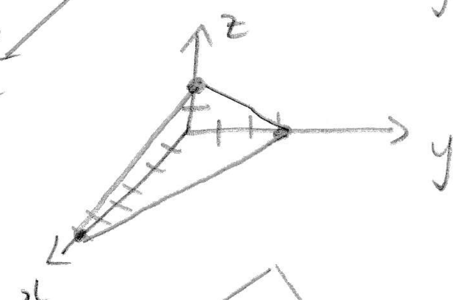
Let $x=0$ and $z=0$ $\quad 2y=6$

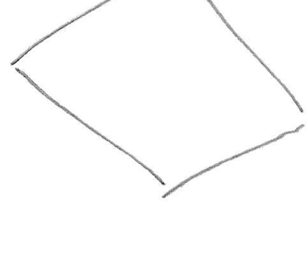
$y=3$

$(0,3,0)$

Similarly $\quad (6,0,0)$







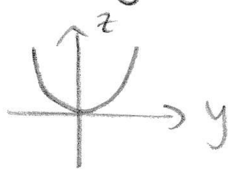
Surface is an infinite plane containing the triangle

Method works when degrees $=1$

Ex: Graph $z = x^2 + y^2$ ← surface in 3D

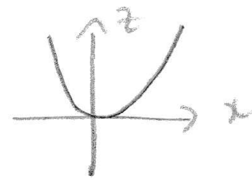
Let $x = 0$:

$z = y^2$



Let $y = 0$

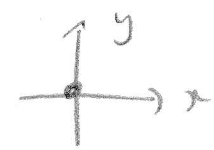
$z = x^2$

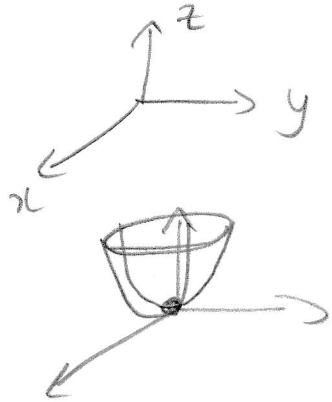


Let $z = 0$

$x^2 + y^2 = 0$

$(x, y) = 0$

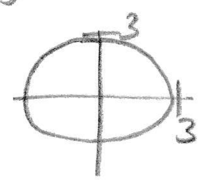




Method works when some degrees $\neq 1$

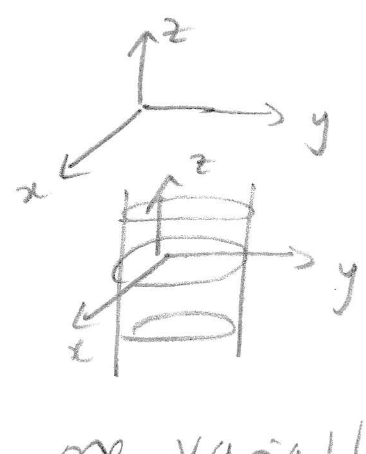
Ex: Graph $x^2 + y^2 = 9$ in 3D

In 2D:



circle

In 3D:

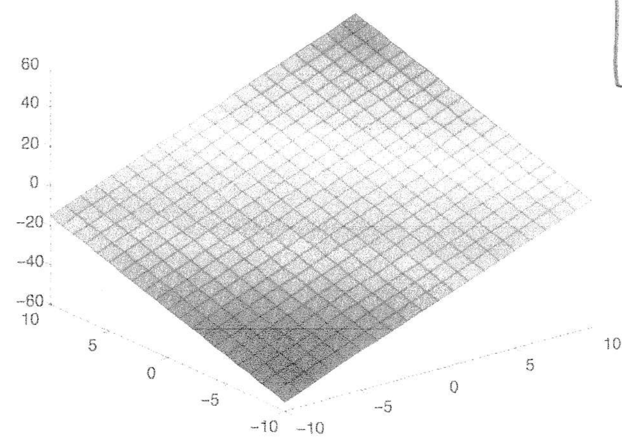


cylinder

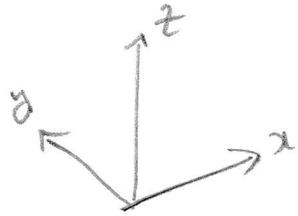
Method works when one variable is missing from equation

HANDOUT

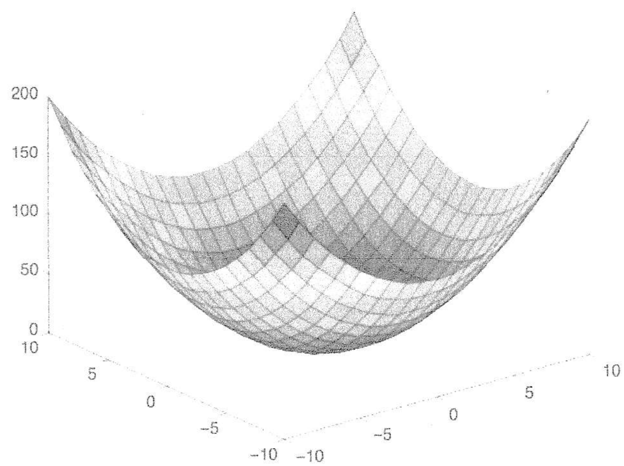
$z = 3x + 2y - 5$



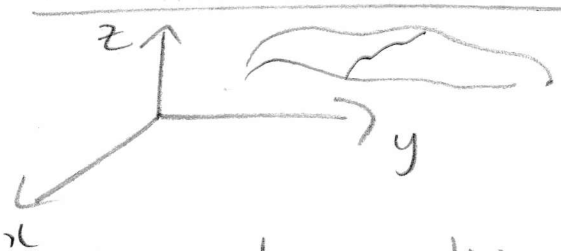
Note:



$z = x^2 + y^2$



Partial Derivatives



← surface in 3D

Take a slice parallel to x-axis

← slope of this line is $\frac{\partial z}{\partial x}$

"partial derivative of z
with respect to x"

Notation: $\frac{\partial z}{\partial x}$ or $\frac{\delta z}{\delta x}$ or z_x

Ex: $f(x,y) = x^2 + 2xy + 6y$

$\frac{\partial f}{\partial x}$: x is the variable (y is constant)

$$\frac{\partial f}{\partial x} = 2x + 2y + 0$$
$$= 2x + 2y$$

$\frac{\partial f}{\partial y}$: y is the variable (x is constant)

$$\frac{\partial f}{\partial y} = 0 + 2x + 6$$
$$= 2x + 6$$

Ex: $f(x,y) = \cos xy$

$$\frac{\partial f}{\partial x} = -\sin xy\,(y)$$
$$= -y\sin xy$$

$$\frac{\partial f}{\partial y} = -\sin xy\,(x)$$
$$= -x\sin xy$$

Ex: $z = \dfrac{e^{6x+y}}{x^2+7}$

Find $\left.\dfrac{\partial z}{\partial x}\right|_{(0,0,\frac{1}{7})}$

Quotient Rule

$$\frac{\partial z}{\partial x} = \frac{(x^2+7)(6e^{6x+y}) - e^{6x+y}(2x)}{(x^2+7)^2}$$

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=0\\y=0}} = \frac{7(6e^0) - e^0(0)}{7^2}$$

$$= \frac{42}{49} \text{ or } \frac{6}{7}$$

$\underline{Ex}$: $z = x^2 \cos 4y$. Find:

a) $\frac{\partial^2 z}{\partial x^2}$

$$\boxed{\frac{\partial z}{\partial x} = 2x \cos 4y}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$
$$= \frac{\partial}{\partial x}(2x \cos 4y)$$
$$= 2 \cos 4y$$

b) $\frac{\partial^2 z}{\partial x \partial y}$

$$\boxed{\begin{aligned}\frac{\partial z}{\partial y} &= x^2(-4 \sin 4y) \\ &= -4x^2 \sin 4y\end{aligned}}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial}{\partial x}(-4x^2 \sin 4y)$$
$$= -8x \sin 4y$$

c) $\left.\frac{\partial^2 z}{\partial y^2}\right|_{(3, \frac{\pi}{12})}$

$$\frac{\partial z}{\partial y} = -4x^2 \sin 4y$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial y}\left(-4x^2 \sin 4y\right) = -4x^2(4 \cos 4y)$$
$$= -16x^2 \cos 4y$$

$$\left.\frac{\partial^2 z}{\partial y^2}\right|_{\substack{x=3\\ y=\frac{\pi}{12}}} = -16(9)\left(\cos\frac{\pi}{3}\right)$$

$$= -144\left(\frac{1}{2}\right)$$

$$= -72$$