28.4 Basic Trig Integration

$\frac{d}{dx}[\sin x] = \cos x$

$$\boxed{\begin{aligned}\int \cos x\,dx &= \sin x + C\\ \int \sin x\,dx &= -\cos x + C\end{aligned}}$$

Know these

$\int \sec^2 x\,dx = \tan x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \tan x\,dx = \ln|\sec x| + C$

$\int \cot x\,dx = -\ln|\csc x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

Refer to FS

Ex: $\int \cos 4x\,dx$

$= \frac{1}{4}\int \cos u\,du$

$= \frac{1}{4}\sin u + C$

$= \frac{\sin 4x}{4} + C$

$u = 4x$
$du = 4dx$
$dx = \frac{du}{4}$

Shortcuts: $\int \sin 3x\,dx = -\frac{\cos 3x}{3} + C$

$\int \sec 7x \tan 7x\,dx = \frac{\sec 7x}{7} + C$ etc.

Ex: a) $\int x \sec(x^2)\tan(x^2)\,dx$

$u = x^2$
$du = 2x\,dx$
$x\,dx = \frac{du}{2}$

$= \frac{1}{2}\int \sec u \tan u\,du$

$= \frac{1}{2}\sec u + C$

$= \frac{1}{2}\sec(x^2) + C$

b) $\int \frac{\tan x}{\cos x}\,dx$

$= \int \sec x \tan x\,dx$

$= \sec x + C$

c) $\int \frac{1}{\sin^2 x}\,dx$

$= \int \csc^2 x\,dx$

$= -\cot x + C$

d) $\int_0^{\sqrt{\frac{\pi}{2}}} 3x \tan x^2\,dx$

$u = x^2$
$du = 2x\,dx$
$x\,dx = \frac{du}{2}$

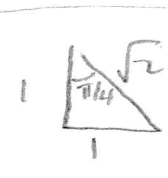
When $x = 0$ $u = 0$
$x = \sqrt{\frac{\pi}{2}}$ $u = \frac{\pi}{4}$

$= \frac{3}{2}\int_0^{\frac{\pi}{4}} \tan u\,du$

$= \frac{3}{2}\Big[\ln|\sec u|\Big]_0^{\pi/4}$

$= \frac{3}{2}\left[\ln(\sqrt{2}) - \ln(1)\right]$

$= \frac{3}{2}\ln\sqrt{2}$



$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\cos 0 = 1$

$\sec\frac{\pi}{4} = \sqrt{2}$ $\sec 0 = 1$

Ex: $\displaystyle\int_0^{\frac{\pi}{12}} \frac{dx}{\cos^2 2x}$

$= \displaystyle\int_0^{\frac{\pi}{12}} \sec^2 2x \, dx$

$u = 2x$
$du = 2dx$
$dx = \frac{du}{2}$
When $x = 0$ $u = 0$
$x = \frac{\pi}{12}$ $u = \frac{\pi}{6}$

$= \frac{1}{2}\displaystyle\int_0^{\frac{\pi}{6}} \sec^2 u \, du$

$= \frac{1}{2}\left[\tan u\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left[\frac{1}{\sqrt{3}} - 0\right)$

$= \frac{1}{2\sqrt{3}}$ " $\frac{\sqrt{3}}{6}$

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$
$\tan 0 = 0$

Ex: $\displaystyle\int \frac{1+\cos x}{\sin x} dx$ DIVIDE

$= \displaystyle\int\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right) dx$

$= \displaystyle\int(\csc x + \cot x) dx$

$= -\ln|\csc x + \cot x| - \ln|\csc x| + C$

Ex: $\displaystyle\int \csc 2x(\sin 2x + \cot 2x) dx$ MULT.

$= \displaystyle\int(1 + \csc 2x \cot 2x) dx$

$= x - \frac{\csc 2x}{2} + C$

$\underline{Ex}$: $\int \sqrt{\tan^2 7x + 1}\, dx$

Hint: Use $1 + \tan^2\theta = \sec^2\theta$

$= \int \sqrt{\sec^2 7x}\, dx$

$= \int \sec 7x \, dx$

$= \frac{1}{7} \ln|\sec 7x + \tan 7x| + C$

$\underline{Ex}$: $\int \frac{dx}{1-\sin x}$

$= \int \frac{1}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x}\, dx$ Trick

$= \int \frac{1+\sin x}{1-\sin^2 x}\, dx$

$= \int \frac{1+\sin x}{\cos^2 x}\, dx$ $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \right) dx$

$= \int (\sec^2 x + \sec x \tan x)\, dx$

$= \tan x + \sec x + C$