28.3 The Exponential Form

$$\frac{d}{du}[e^u] = e^u$$

$$\boxed{\int e^u\,du = e^u + C}$$

Ex: $\int x e^{2x^2}\,dx$

$$= \frac{1}{4}\int e^u\,du$$
$$= \frac{1}{4}e^u + C$$
$$= \frac{1}{4}e^{2x^2} + C$$

$u = 2x^2$
$du = 4x\,dx$
$x\,dx = \frac{du}{4}$

Ex: $\int e^{4x}\,dx$

$u = 4x$
$du = 4\,dx$
$dx = \frac{du}{4}$

$$= \frac{1}{4}\int e^u\,du$$
$$= \frac{1}{4}e^u + C$$
$$= \frac{e^{4x}}{4} + C$$

Shortcut

$$\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$$
$(k \neq 0)$

Ex: $\displaystyle\int \frac{dx}{\sqrt{e^x}}$

$$= \int (e^x)^{-1/2}\, dx$$

$$= \int e^{-x/2}\, dx$$

$$= -2\int e^u\, du$$

$$= -2e^u + C$$

$$= -2e^{-x/2} + C$$

Let $u = -x/2$

$du = -\tfrac{1}{2}\, dx$

$-2\, du = dx$

Ex: $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

$$= \int e^{\sqrt{x}} \left(x^{-1/2}\, dx \right)$$

$$= 2\int e^u\, du$$

$$= 2e^u + C$$

$$= 2e^{\sqrt{x}} + C$$

$u = \sqrt{x}$

$du = \tfrac{1}{2} x^{-1/2}\, dx$

$2\, du = x^{-1/2}\, dx$

$\frac{dx}{\sqrt{x}} = 2\, du$

Ex: $\displaystyle\int e^{6x}\left(e^{2x} + e^{-6x}\right) dx$ EXPAND

e^0

$$= \int \left(e^{8x} + 1\right) dx$$

$$= \frac{e^{8x}}{8} + x + C$$

Ex: $\int \frac{7e^{6x}}{e^{2x}}\,dx$ DIVIDE

$= \int 7e^{4x}\,dx$

$= \frac{7e^{4x}}{4} + C$

Ex: $\int_{-2}^{-1} \frac{dx}{x^2 e^{1/x}}$

$= \int_{-2}^{-1} e^{-1/x}\,\frac{dx}{x^2}$

$= \int_{1/2}^{1} e^u\,du = [e^u]_{1/2}^{1}$

$= e^1 - e^{1/2}$

$u = -1/x = -x^{-1}$

$du = x^{-2}\,dx$

When $x = -2$ $u = \frac{1}{2}$

$x = -1$ $u = 1$

Ex: $\int_0^{\pi/6} \cos 3\theta\, e^{\sin 3\theta}\,d\theta$

$= \frac{1}{3}\int_0^1 e^u\,du$

$= \frac{1}{3}[e^u]_0^1$

$= \frac{1}{3}[e^1 - e^0]$ $= \frac{1}{3}(e-1)$

$u = \sin 3\theta$

$du = 3\cos 3\theta\,d\theta$

$\frac{1}{3}du = \cos 3\theta\,d\theta$

When $x = 0$ $u = 0$

$x = \frac{\pi}{6}$ $u = \sin\frac{\pi}{2} = 1$

Ex: $\int_0^{\frac{\pi}{2}} (\sin 2x) e^{\sin^2 x}\, dx$

Hint: $\sin 2x = 2\sin x \cos x$

$u = \sin^2 x$
$du = 2\sin x \cdot \cos x \, dx$
$du = \sin 2x \, dx$
When $x = 0$ $u = (\sin 0)^2 = 0$
$x = \frac{\pi}{2}$ $u = (\sin \frac{\pi}{2})^2 = 1$

$= \int_0^1 e^u \, du$

$= [e^u]_0^1$

$= e^1 - e^0$

$= e - 1$

Ex: $\int \sqrt{e^{4y} + e^{6y}} \, dy$

$= \int \sqrt{e^{4y}(1+e^{2y})} \, dy = \int \sqrt{e^{4y}} \sqrt{(1+e^{2y})} \, dy$

$= \int e^{2y} \sqrt{1+e^{2y}} \, dy$

$u = 1 + e^{2y}$
$du = 2e^{2y} \, dy$
$\frac{1}{2} du = e^{2y} \, dy$

$= \frac{1}{2} \int \sqrt{u} \, du$

$= \frac{1}{\cancel{2}} \left(\frac{\cancel{2}}{3} u^{3/2} \right) + C$

$= \frac{1}{3} (1 + e^{2y})^{3/2} + C$