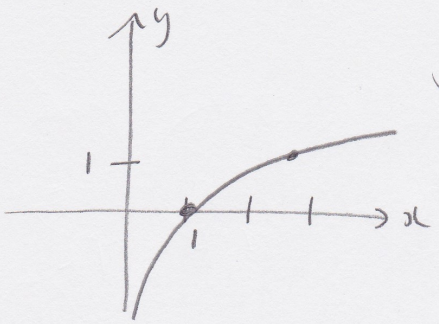
28.2 The Logarithmic Form



$y = \ln x$

$\ln 1 = 0$
$\ln e = 1$

$\ln x$ is only defined for $x > 0$

$$\int \frac{du}{u} = \ln|u| + C$$

Contrast with Section 28.1

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

Ex: Evaluate $\int \frac{3x}{4-2x^2}\,dx$

$u = 4-2x^2$

$du = -4x\,dx$

$-\frac{1}{4}du = x\,dx$

$-\frac{3}{4}du = 3x\,dx$

$= -\frac{3}{4}\int \frac{du}{u}$

$= -\frac{3}{4}\ln|u| + C$

$= -\frac{3}{4}\ln|4-2x^2| + C$

Ex: Evaluate $\int \frac{6\,dx}{(\sin^{-1}x)\sqrt{1-x^2}}$

$u = \sin^{-1}x$

$du = \frac{dx}{\sqrt{1-x^2}}$

$= 6\int \frac{du}{u}$

$= 6\ln|u| + C$

$= 6\ln|\sin^{-1}x| + C$

Ex: Evaluate $\int \frac{e^{7x}}{1+2e^{7x}}\,dx$

$u = 1+2e^{7x}$

$du = 14e^{7x}\,dx$

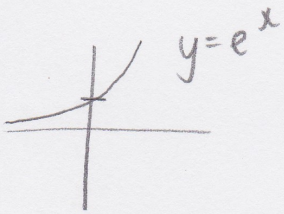
$\frac{du}{14} = e^{7x}\,dx$

$= \frac{1}{14}\int \frac{du}{u}$

$= \frac{1}{14}\ln|u| + C$

$= \frac{1}{14}\ln|1+2e^{7x}| + C$

OR $\frac{1}{14}\ln(\underbrace{1+2e^{7x}}_{>0}) + C$



Ex: Evaluate $\int \frac{dx}{x\ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \ln|\ln x| + C$

Ex: Evaluate $\int \frac{x^2 - x}{x^2}\,dx$

$$= \int \left(1 - \frac{1}{x}\right) dx$$

$$= x - \ln|x| + C$$

RECAP Ln Rules

$$\ln a + \ln b = \ln(ab)$$

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

$$n \ln a = \ln a^n$$

Ex: Evaluate $\int \tan x\,dx$

$$= \int \frac{\sin x\,dx}{\cos x}$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$= -\int \frac{du}{u}$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

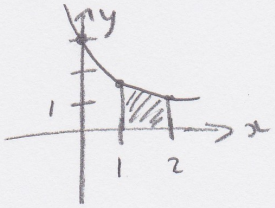
OR $\ln|\cos x|^{-1} + C$

$n \ln a = \ln a^n$

$$= \ln \frac{1}{|\cos x|} + C$$

$$= \ln|\sec x| + C$$

Ex: Area under $y = \frac{3}{x+1}$ from $x=1$ to $x=2$?



$$A = \int_1^2 \frac{3}{x+1}\,dx$$

$u = x+1$
$du = dx$
When $x=1$, $u=2$
$x=2$, $u=3$

$$= 3\int_2^3 \frac{du}{u}$$

$$= 3\left[\ln|u|\right]_2^3$$

$$= 3[\ln|3| - \ln|2|]$$

$$= 3[\ln 3 - \ln 2]$$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

OR $3\ln\frac{3}{2}$

Ex: Evaluate $\int_{\sqrt{2}}^{2} \frac{du}{u}$

$$= \left[\ln|u|\right]_{\sqrt{2}}^{2}$$

$$= \ln|2| - \ln|\sqrt{2}|$$

$$= \ln 2 - \ln\sqrt{2}$$

OR $\ln 2 - \frac{1}{2}\ln 2$

$$= \frac{\ln 2}{2}$$

$\ln a^n = n\ln a$
$\ln\sqrt{2} = \frac{1}{2}\ln 2$