Math 193

Integration (Theory)

Differential Equations (Theory and Applications)

e.g. spring-mass systems
deflection of beams etc.

Statistics (Applications)

28.1 General Power Formula

RECAP

$f(x)$	$f'(x)$
x^{-2}	$-2x^{-3}$
$x^{4/3}$	$\frac{4}{3}x^{1/3}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\ln 2x+1$	$\frac{2}{2x+1}$
$\ln g(x)$	$\frac{g'(x)}{g(x)}$
e^{7x}	$7e^{7x}$
$e^{g(x)}$	$g'(x) \cdot e^{g(x)}$

Hints on formula sheet

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int u^{-3} du = -\frac{1}{2}u^{-2} + C$$

$$\int u^{2/3} du = \frac{3}{5}u^{5/3} + C \quad \text{etc.}$$

Ex: $\int \sqrt{\sin x}\cos x\, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$= \int \sqrt{u}\, du$

$= \int u^{1/2} du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(\sin x)^{3/2} + C$ or $\frac{2}{3}\sin^{\frac{3}{2}} x + C$

Note: Can't integrate $\int \sqrt{\sin x}\, dx$

Ex: Evaluate $\int_0^{\pi/16} \sin 4x \cos 4x \, dx$

$u = \sin 4x$

$du = 4\cos 4x \, dx$

$\frac{du}{4} = \cos 4x \, dx$

When $x = 0$, $u = \sin 0 = 0$

$x = \frac{\pi}{16}$, $u = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$= \int_0^{\frac{1}{\sqrt{2}}} \frac{u \, du}{4}$$

$$= \frac{1}{4} \int_0^{\frac{1}{\sqrt{2}}} u \, du$$

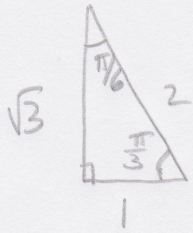
$$= \frac{1}{4} \left[\frac{u^2}{2} \right]_0^{\frac{1}{\sqrt{2}}}$$

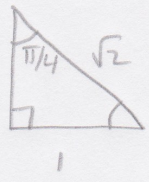
$$= \frac{1}{4} \left[\frac{1}{2} \left(\frac{1}{\sqrt{2}} \right)^2 - 0 \right]$$

$$= \frac{1}{4} \left[\frac{1}{4} - 0 \right]$$

$$= \frac{1}{16}$$

RECAP

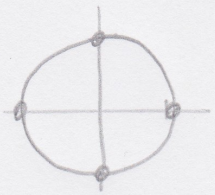




SOHCAHTOA

$$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sec\frac{\pi}{4} = \frac{H}{A} = \frac{\sqrt{2}}{1} = \sqrt{2}$$



$\sin\pi = 0$ (y-value)

$\cos\pi = -1$ (x-value)

$\sin\frac{3\pi}{2} = -1$

Ex: Evaluate $\int \sec^4 t \cdot \tan t \, dt$

$u = \sec t$

$du = \sec t \tan t \, dt$

$$= \int \sec^3 t \cdot \sec t \tan t \, dt$$

$$= \int u^3 \, du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{1}{4}\sec^4 t + C$$

Ex: Evaluate $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}}\,dx$

$u = \cos^{-1} x$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx$

$-du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= -\int u\,du$

$= -\frac{u^2}{2} + C$

$= -\frac{1}{2}(\cos^{-1} x)^2 + C$

Ex: Evaluate $\int \frac{1}{x(\ln 2x)^3}\,dx$

$u = \ln 2x$

$du = \frac{2}{2x}\,dx$

$du = \frac{dx}{x}$

$= \int \frac{du}{u^3}$

$= \int u^{-3}\,du$

$= -\frac{1}{2}u^{-2} + C$

$= -\frac{1}{2}(\ln 2x)^{-2} + C$

Ex: Evaluate $\int \frac{1+3e^{-2x}}{e^{2x}}\,dx$

$u = 1 + 3e^{-2x}$

$du = -6e^{-2x}\,dx$

$-\frac{du}{6} = e^{-2x}\,dx = \frac{dx}{e^{2x}}$

$= -\frac{1}{6}\int u\,du$

$= -\frac{1}{6}\left(\frac{1}{2}u^2\right) + C$

$= -\frac{1}{12}\left(1+3e^{-2x}\right)^2 + C$

Ex: Evaluate $\int \sqrt[3]{(1+e^{2x})(1-e^{2x})}\,e^{4x}\,dx$

$= \int \sqrt[3]{1-e^{4x}}\,e^{4x}\,dx$

$u = 1 - e^{4x}$

$du = -4e^{4x}\,dx$

$-\frac{du}{4} = e^{4x}\,dx$

$= -\frac{1}{4}\int \sqrt[3]{u}\,du$

$= -\frac{1}{4}\int u^{1/3}\,du$

$= -\frac{1}{4}\left(\frac{3}{4}u^{4/3}\right) + C$

$= -\frac{3}{16}\left(1 - e^{4x}\right)^{4/3} + C$

Ex: Evaluate $\int \frac{\tan^{-1} 4x}{1+16x^2}\,dx$

$u = \tan^{-1} 4x$

$du = \frac{1}{1+(4x)^2} \cdot 4\,dx$

$du = \frac{4}{1+16x^2}\,dx$

$\frac{du}{4} = \frac{dx}{1+16x^2}$

$= \frac{1}{4}\int u\,du$

$= \frac{1}{8}u^2 + C$

$= \frac{1}{8}(\tan^{-1} 4x)^2 + C$

Ex: Slope of a curve is $(1+\tan 2x)^2 \sec^2 2x$ and curve passes through the point $(0,4)$. Find the equation of the curve y.

slope $\frac{dy}{dx} = (1+\tan 2x)^2 \sec^2 2x$

$y = \int (1+\tan 2x)^2 \sec^2 2x\,dx$

$$u = 1 + \tan 2x$$
$$du = 2\sec^2 2x\, dx$$
$$\frac{du}{2} = \sec^2 2x\, dx$$

$$y = \frac{1}{2}\int u^2\, du$$
$$y = \frac{1}{6}u^3 + C$$
$$\boxed{y = \frac{1}{6}(1 + \tan 2x)^3 + C}$$

Sub $x = 0$, $y = 4$:
$$4 = \frac{1}{6}(1 + 0)^3 + C$$
$$4 - \frac{1}{6} = C$$
$$\frac{24}{6} - \frac{1}{6} = C$$
$$C = \frac{23}{6}$$

$$y = \frac{1}{6}(1 + \tan 2x)^3 + \frac{23}{6}$$