

1. [3 marks] Evaluate  $\int \frac{e^{4x}}{\csc e^{4x}} dx$

$$= \int e^{4x} \sin e^{4x} dx$$

$$= \frac{1}{4} \int \sin u du$$

$$= -\frac{1}{4} \cos u + C$$

$$= -\frac{1}{4} \cos e^{4x} + C$$

$$\begin{aligned} u &= e^{4x} \\ du &= 4e^{4x} dx \\ \frac{du}{4} &= e^{4x} dx \end{aligned}$$

2. [3 marks] Evaluate  $\int \frac{\sin 3x}{1+\cos^2 3x} dx$

$$= -\frac{1}{3} \int \frac{du}{1+u^2}$$

$$= -\frac{1}{3} \tan^{-1} u + C$$

$$= -\frac{1}{3} \tan^{-1} (\cos 3x) + C$$

$$\begin{aligned} u &= \cos 3x \\ du &= -3 \sin 3x dx \\ -\frac{du}{3} &= \sin 3x dx \end{aligned}$$

3. [3 marks] Evaluate  $\int_0^1 \frac{dx}{(1+x^2)\sqrt{\tan^{-1}x}}$

$$\begin{aligned} &= \int_0^{\pi/4} \frac{du}{\sqrt{u}} \\ &= 2\sqrt{u} \Big|_0^{\pi/4} \\ &= 2\sqrt{\frac{\pi}{4}} - 2\sqrt{0} \\ &= 2\sqrt{\frac{\pi}{4}} \text{ or } \sqrt{\pi} \text{ or } 1.77 \end{aligned}$$

$$\begin{aligned} u &= \tan^{-1}x \\ du &= \frac{dx}{1+x^2} \\ x=0 &\rightarrow u = \tan^{-1}0 = 0 \\ x=1 &\rightarrow u = \tan^{-1}1 = \frac{\pi}{4} \end{aligned}$$

4. [2 marks] Evaluate  $\int \frac{e^y}{1+e^y} dy$

$$\begin{aligned} &= \int \frac{du}{u} \\ &= \ln|u| + c \\ &= \ln|1+e^y| + c \end{aligned}$$

$$\begin{aligned} u &= 1+e^y \\ du &= e^y dy \end{aligned}$$

5. [4 marks] Use Integration by Parts to evaluate  $\int x^2 e^{-2x} dx$

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(+)	$x^2$	$e^{-2x}$
(-)	$2x$	$-\frac{1}{2}e^{-2x}$
(+)	$2$	$\frac{1}{4}e^{-2x}$
	$0$	$-\frac{1}{8}e^{-2x}$

Answer:  $-\frac{x^2}{2}e^{-2x} - \frac{2x}{4}e^{-2x} - \frac{2}{8}e^{-2x} + C$   
 or  $-\left(\frac{x^2}{2} + \frac{x}{2} + \frac{1}{4}\right)e^{-2x} + C$

6. [2 marks] Let  $z = x^2y^3 + y \ln x$ . Find:

a)  $\frac{\partial z}{\partial y} = 3x^2y^2 + \ln x$

b)  $\frac{\partial^2 z}{\partial x \partial y} = \frac{d}{dx} \left( \frac{\partial z}{\partial y} \right)$   
 $= 6xy^2 + \frac{1}{x}$

7. [4 marks] Use Partial Fractions to evaluate  $\int \frac{4x-5}{(x+1)(x-4)} dx$

$$\frac{4x-5}{(x+1)(x-4)} = \frac{A}{x+1} + \frac{B}{x-4}$$

$$4x-5 = A(x-4) + B(x+1)$$

$$\text{Sub } x=4: \quad 11 = 5B$$

$$B = \frac{11}{5}$$

$$\text{Sub } x=-1: \quad -9 = -5A$$

$$A = \frac{9}{5}$$

$$\text{Integral} = \int \left( \frac{9}{5} \cdot \frac{1}{x+1} + \frac{11}{5} \cdot \frac{1}{x-4} \right) dx$$

$$= \frac{9}{5} \ln|x+1| + \frac{11}{5} \ln|x-4| + C$$

8. [4 marks] Evaluate  $\int_0^2 \int_x^3 (x + 2y) dy dx$

$$= \int_0^2 [xy + y^2]_{y=x}^{y=3} dx$$

$$= \int_0^2 (3x + 9 - x^2 - x^2) dx$$

$$= \int_0^2 (3x + 9 - 2x^2) dx$$

$$= \left[ \frac{3x^2}{2} + 9x - \frac{2x^3}{3} \right]_0^2$$

$$= 6 + 18 - \frac{16}{3}$$

$$= \frac{56}{3}$$