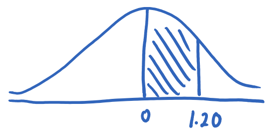
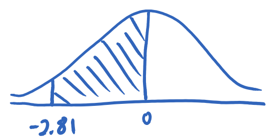
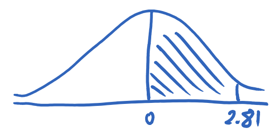
7 The Normal Distribution

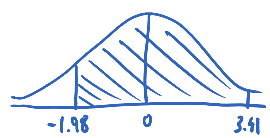
1\. a) $P(0 \leq z \leq 1.20) =$ 

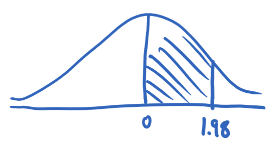
$= 0.3849$

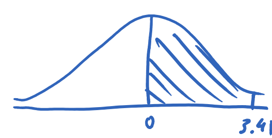
b) $P(-2.81 \leq z \leq 0) =$ 

$=$ 

$= 0.4975$

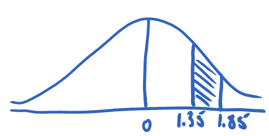
c) $P(-1.98 \leq z \leq 3.41) =$ 

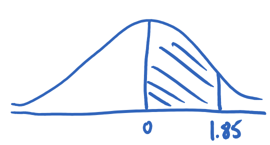
$=$ 

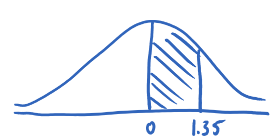
$+$ 

$= 0.4761 + 0.4997$

$= 0.9758$

d) $P(1.35 \leq z \leq 1.85) =$ 

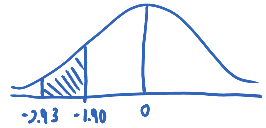
$=$ 

$-$ 

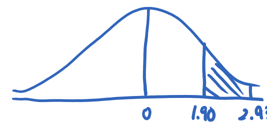
$= 0.4678 - 0.4115$

$= 0.0563$

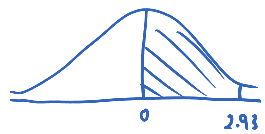
e) $P(-2.93 \leq z \leq -1.90) =$



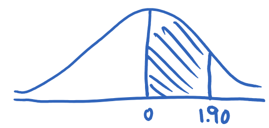
$=$



$=$



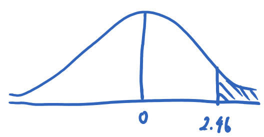
$-$



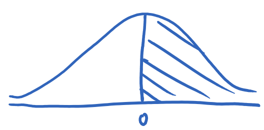
$= 0.4983 - 0.4713$

$= 0.0270$

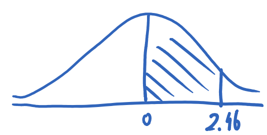
f) $P(z \geq 2.46) =$



$=$



$-$

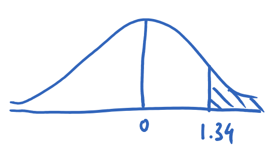


$= 0.5 - 0.4931$

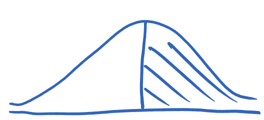
$= 0.0069$

g) $P(z \leq -1.34) =$

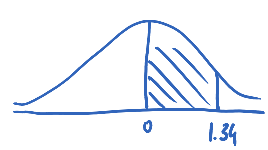
$=$



$=$

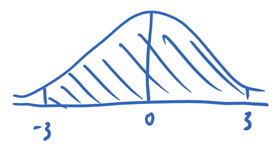


$-$



$= 0.5 - 0.4099 = 0.0901$

2. $P(\mu - 3\sigma \le X \le \mu + 3\sigma) = P(-3 \le z \le 3)$

$=$ 

$= 2(0.4987)$

$= 0.9974$

Note: $x = \mu - 3\sigma \Rightarrow z = \dfrac{(\mu - 3\sigma) - \mu}{\sigma} = -3$

$x = \mu + 3\sigma \Rightarrow z = \dfrac{(\mu + 3\sigma) - \mu}{\sigma} = 3$

3. a) we want $P(X < 1500)$

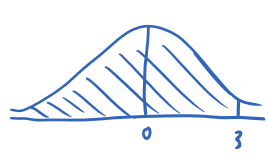
$z = \dfrac{x - \mu}{\sigma} = \dfrac{1500 - 1050}{150} = 3$

[Note: we don't need to specify $X \ge 0$ since

$x = 0 \Rightarrow z = \dfrac{0 - 1050}{150} = -7$

so the entire z-curve satisfies $X \ge 0$]

$P(X < 1500) = P(z < 3)$

$=$  $= 0.5 + 0.4987 = 0.9987$

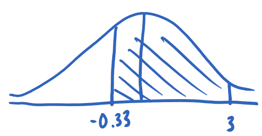
b) we want $P(1000 < X < 1500)$

$x = 1000 \Rightarrow z = \frac{1000 - 1050}{150} = -0.33$

$x = 1500 \Rightarrow z = 3$

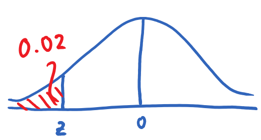
$P(1000 < X < 1500) = P(-0.33 < z < 3)$

=



$= 0.1293 + 0.4987$
$= 0.6280$

c) we want x such that $P(X \leq x) = 0.02$



reverse look-up: area $= 0.5 - 0.02 = 0.48 \Rightarrow z = 2.05$
but $z < 0$ so $z = -2.05$

$z = \frac{x - \mu}{\sigma}$

$-2.05 = \frac{x - 1050}{150}$

$-307.5 = x - 1050$

$x = 742.5$

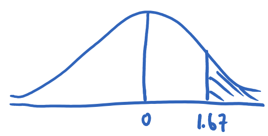
The smallest 2% of all droplets have size under 742.5 μm.

4. we want $P(X > 300)$

$$z = \frac{x - \mu}{\sigma} = \frac{300 - 250}{30} = 1.67$$

$$P(X > 300) = P(z > 1.67)$$

=



$$= 0.5 - 0.4525$$
$$= 0.0475$$

5. a) we want $P(100 < X < 120)$

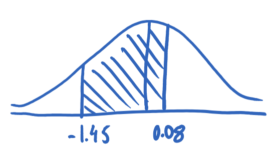
$$z = \frac{x - \mu}{\sigma}$$

$$x = 100 \Rightarrow z = \frac{100 - 119}{13.1} = -1.45$$

$$x = 120 \Rightarrow z = \frac{120 - 119}{13.1} = 0.08$$

$$P(100 < X < 120) = P(-1.45 < z < 0.08)$$

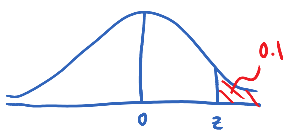
=



$$= 0.4265 + 0.0319$$
$$= 0.4584$$

45.84%

b) we want x such that $P(X \geq x) = 0.1$



reverse look-up:
area $= 0.5 - 0.1 = 0.4 \Rightarrow z = 1.28$

$$z = \frac{x - \mu}{\sigma}$$

$$1.28 = \frac{x - 119}{13.1}$$

$$16.768 = x - 119$$

$$x = 135.8$$

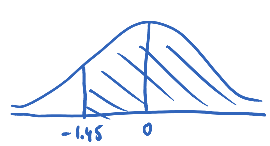
The fastest 10% of all speeds are above 135.8 km/h

c) we want $P(X > 100)$

$$z = \frac{x-\mu}{\sigma} = \frac{100 - 119}{13.1} = -1.45$$

$$P(X > 100) = P(z > -1.45)$$

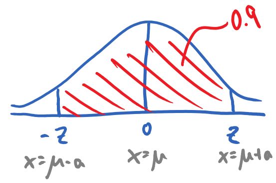
$=$



$$= 0.4265 + 0.5$$
$$= 0.9265$$

92.65% exceeded the posted limit

d) we want a such that $P(\mu - a < X < \mu + a) = 0.9$



reverse look-up:

area $= \frac{1}{2}(0.9) = 0.45 \Rightarrow z = 1.64$

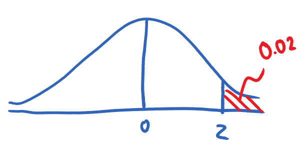
$z = 1.65$ is also correct

$$z = \frac{x - \mu}{\sigma}$$

$$1.64 = \frac{(\mu + a) - \mu}{13.1}$$

$$21.484 = a$$

6. we want μ given $P(X < 4) = 0.02$



reverse look-up:

area $= 0.5 - 0.02 = 0.48 \Rightarrow z = 2.05$

$$z = \frac{x - \mu}{\sigma}$$

$$2.05 = \frac{4 - \mu}{0.04}$$

$$0.082 = 4 - \mu$$

$\mu = 4.082$ ounces