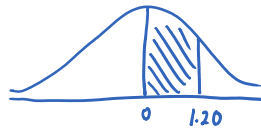


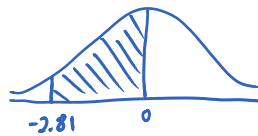
7 The Normal Distribution

1. a) $P(0 \leq z \leq 1.20) =$

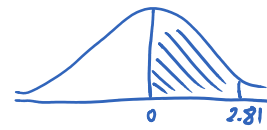


$$= 0.3849$$

b) $P(-2.81 \leq z \leq 0) =$



=

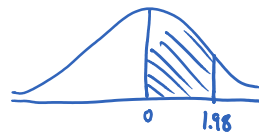


$$= 0.4975$$

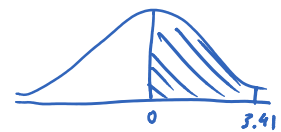
c) $P(-1.98 \leq z \leq 3.41) =$



=



+



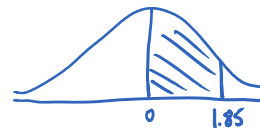
$$= 0.4761 + 0.4997$$

$$= 0.9758$$

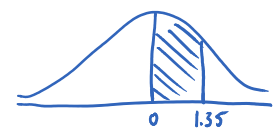
d) $P(1.35 \leq z \leq 1.85) =$



=



-



$$= 0.4678 - 0.4115$$

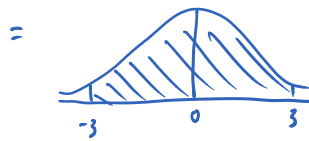
$$= 0.0563$$

$$\begin{aligned}
 e) \quad P(-2.93 \leq z \leq -1.90) &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area between } z = -2.93 \text{ and } z = -1.90 \text{ is shaded.} \\ \text{---} \\ -2.93 \quad -1.90 \quad 0 \end{array} \\
 &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 1.90 \text{ is shaded.} \\ \text{---} \\ 0 \quad 1.90 \quad 2.93 \end{array} \\
 &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 2.93 \text{ is shaded.} \\ \text{---} \\ 0 \quad 2.93 \end{array} - \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 1.90 \text{ is shaded.} \\ \text{---} \\ 0 \quad 1.90 \end{array} \\
 &= 0.4983 - 0.4713 \\
 &= 0.0270
 \end{aligned}$$

$$\begin{aligned}
 f) \quad P(z \geq 2.46) &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 2.46 \text{ is shaded.} \\ \text{---} \\ 0 \quad 2.46 \end{array} \\
 &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 2.46 \text{ is shaded.} \\ \text{---} \\ 0 \quad 2.46 \end{array} - \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 2.46 \text{ is shaded.} \\ \text{---} \\ 0 \quad 2.46 \end{array} \\
 &= 0.5 - 0.4931 \\
 &= 0.0069
 \end{aligned}$$

$$\begin{aligned}
 g) \quad P(z \leq -1.34) &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the left of } z = -1.34 \text{ is shaded.} \\ \text{---} \\ -1.34 \quad 0 \end{array} \\
 &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 1.34 \text{ is shaded.} \\ \text{---} \\ 0 \quad 1.34 \end{array} \\
 &= \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 1.34 \text{ is shaded.} \\ \text{---} \\ 0 \quad 1.34 \end{array} - \begin{array}{c} \text{Normal distribution curve with mean 0. The area to the right of } z = 1.34 \text{ is shaded.} \\ \text{---} \\ 0 \quad 1.34 \end{array} \\
 &= 0.5 - 0.4099 = 0.0901
 \end{aligned}$$

$$2. P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) = P(-3 \leq z \leq 3)$$



$$= 2(0.4987)$$

$$= 0.9974$$

$$\text{Note: } x = \mu - 3\sigma \Rightarrow z = \frac{(\mu - 3\sigma) - \mu}{\sigma} = -3$$

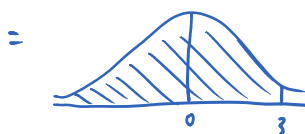
$$x = \mu + 3\sigma \Rightarrow z = \frac{(\mu + 3\sigma) - \mu}{\sigma} = 3$$

$$3. a) \text{ we want } P(X < 1500)$$

$$z = \frac{x - \mu}{\sigma} = \frac{1500 - 1050}{150} = 3$$

Note: we don't need to specify $X \geq 0$ since
 $x = 0 \Rightarrow z = \frac{0 - 1050}{150} = -7$
 so the entire z-curve satisfies $X \geq 0$

$$P(X < 1500) = P(z < 3)$$



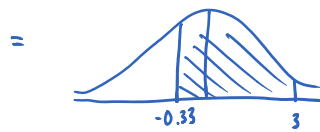
$$= 0.5 + 0.4987 = 0.9987$$

b) we want $P(1000 < X < 1500)$

$$x = 1000 \Rightarrow z = \frac{1000 - 1050}{150} = -0.33$$

$$x = 1500 \Rightarrow z = 3$$

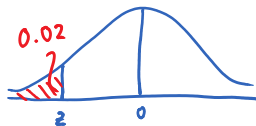
$$P(1000 < X < 1500) = P(-0.33 < z < 3)$$



$$= 0.1293 + 0.4987$$

$$= 0.6280$$

c) we want x such that $P(X \leq x) = 0.02$



reverse look-up: area = $0.5 - 0.02 = 0.48 \Rightarrow z = 2.05$

but $z < 0$ so $z = -2.05$

$$z = \frac{x - \mu}{\sigma}$$

$$-2.05 = \frac{x - 1050}{150}$$

$$-307.5 = x - 1050$$

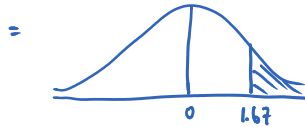
$$x = 742.5$$

The smallest 2% of all droplets have size under $742.5 \mu\text{m}$.

4. we want $P(X > 300)$

$$z = \frac{x - \mu}{\sigma} = \frac{300 - 250}{30} = 1.67$$

$$P(X > 300) = P(z > 1.67)$$



$$= 0.5 - 0.4525$$

$$= 0.0475$$

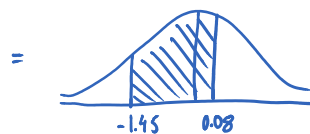
5. a) we want $P(100 < X < 120)$

$$z = \frac{x - \mu}{\sigma}$$

$$x = 100 \Rightarrow z = \frac{100 - 119}{13.1} = -1.45$$

$$x = 120 \Rightarrow z = \frac{120 - 119}{13.1} = 0.08$$

$$P(100 < X < 120) = P(-1.45 < z < 0.08)$$

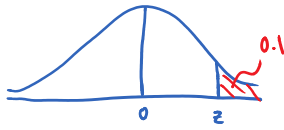


$$= 0.4265 + 0.0319$$

$$= 0.4584$$

45.84%

b) we want x such that $P(X \geq x) = 0.1$



reverse look-up:

$$\text{area} = 0.5 - 0.1 = 0.4 \Rightarrow z = 1.28$$

$$z = \frac{x - \mu}{\sigma}$$

$$1.28 = \frac{x - 119}{13.1}$$

$$16.768 = x - 119$$

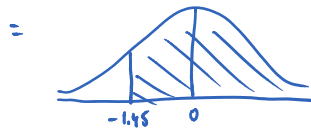
$$x = 135.8$$

The fastest 10% of all speeds are above 135.8 km/h

c) we want $P(X > 100)$

$$z = \frac{x - \mu}{\sigma} = \frac{100 - 119}{13.1} = -1.45$$

$$P(X > 100) = P(z > -1.45)$$

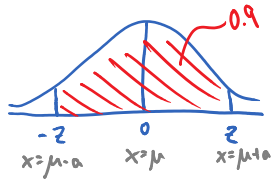


$$= 0.4265 + 0.5$$

$$= 0.9265$$

92.65% exceeded the posted limit

d) we want a such that $P(\mu - a < X < \mu + a) = 0.9$



reverse look-up:

$$\text{area} = \frac{1}{2}(0.9) = 0.45 \Rightarrow z = 1.64$$

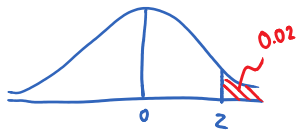
$z = 1.65$ is also correct

$$z = \frac{x - \mu}{\sigma}$$

$$1.64 = \frac{(\mu + a) - \mu}{13.1}$$

$$21.484 = a$$

6. we want μ given $P(X < 4) = 0.02$



reverse look-up:

$$\text{area} = 0.5 - 0.02 = 0.48 \Rightarrow z = 2.05$$

$$z = \frac{x - \mu}{\sigma}$$

$$2.05 = \frac{4 - \mu}{0.04}$$

$$0.082 = 4 - \mu$$

$$\mu = 4.082 \text{ ounces}$$