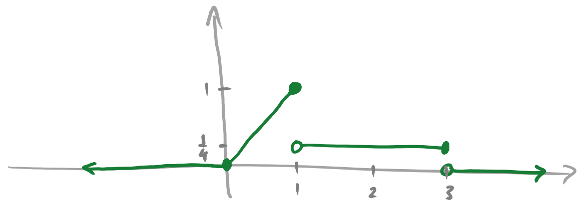
# 6 Continuous Random Variables

1.




a) $P(X=1.5)=0$

b) $$P(0.5<X<1.5)=\int_{0.5}^{1.5} f(x)\,dx$$

$$=\int_{0.5}^{1} x\,dx + \int_{1}^{1.5} \frac{1}{4}\,dx$$

$$=\frac{1}{2}x^2\Big|_{0.5}^{1} + \frac{1}{4}x\Big|_{1}^{1.5}$$

$$=\frac{1}{2}(1^2-0.5^2) + \frac{1}{4}(1.5-1)$$

$$=0.5$$

c) $P(0.5\le X\le 1.5)=P(0.5<X<1.5)=0.5$

d) $$P(X>1.5)=\int_{1.5}^{\infty} f(x)\,dx$$

$$=\int_{1.5}^{3}\frac{1}{4}\,dx$$

$$=\frac{1}{4}x\Big|_{1.5}^{3}$$

$$=\frac{1}{4}(3-1.5)$$

$$=0.375$$

e) $$P(X<0.5) = \int_{-\infty}^{0.5} f(x)\,dx$$
$$= \int_{0}^{0.5} x\,dx$$
$$= \frac{1}{2} x^2 \Big|_0^{0.5}$$
$$= \frac{1}{2}(0.5^2 - 0^2)$$
$$= 0.125$$

f) $$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$$
$$= \int_0^1 x \cdot x\,dx + \int_1^3 x \cdot \frac{1}{4}\,dx$$
$$= \int_0^1 x^2\,dx + \int_1^3 \frac{1}{4} x\,dx$$
$$= \frac{1}{3}x^3 \Big|_0^1 + \frac{1}{8}x^2 \Big|_1^3$$
$$= \frac{1}{3}(1^3 - 0^3) + \frac{1}{8}(3^2 - 1^2)$$
$$= \frac{4}{3} \approx 1.33$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{23}{36}} \approx 0.80$$

$$\sigma^2 = E(X^2) - \mu^2 = \frac{29}{12} - \left(\frac{4}{3}\right)^2 = \frac{23}{36}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$
$$= \int_0^1 x^2\, x\,dx + \int_1^3 x^2\, \frac{1}{4}\,dx$$
$$= \frac{1}{4}x^4 \Big|_0^1 + \frac{1}{12}x^3 \Big|_1^3 = \frac{29}{12}$$

2. a) we need $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$1 = \int_0^2 kx^4\,dx$$

$$1 = \frac{k}{5}x^5 \Big|_0^2$$

$$1 = \frac{k}{5}(2^5 - 0^5)$$

$$5 = 32k$$

$$k = \frac{5}{32}$$

b) $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_0^2 x \cdot \frac{5}{32}x^4\,dx$$

$$= \int_0^2 \frac{5}{32}x^5\,dx$$

$$= \frac{5}{192}x^6 \Big|_0^2$$

$$= \frac{5}{192}(2^6 - 0^6)$$

$$= \frac{5}{3} \approx 1.67$$

c) $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{5}{63}} \approx 0.28$

$$\sigma^2 = E(X^2) - \mu^2 = \frac{20}{7} - \left(\frac{5}{3}\right)^2 = \frac{5}{63}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_0^2 x^2 \frac{5}{32} x^4\, dx$$

$$= \int_0^2 \frac{5}{32} x^6\, dx$$

$$= \frac{5}{224} x^7 \Big|_0^2$$

$$= \frac{5}{224}\left(2^7 - 0^7\right)$$

$$= \frac{20}{7}$$

3. a) $P(X = 0.2) = 0$

b) $P(X < 0.2) = \int_{-\infty}^{0.2} f(x)\, dx$

$$= \int_0^{0.2} \frac{1}{(\ln 2)(x+1)}\, dx$$

$$= \frac{1}{\ln 2} \ln|x+1| \Big|_0^{0.2}$$

$$= \frac{1}{\ln 2}\left(\ln|0.2+1| - \ln|0+1|\right)$$

$$= 0.26$$

c) $$P(X \geq 0.3) = \int_{0.3}^{\infty} f(x)\,dx$$

$$= \int_{0.3}^{1} \frac{1}{(\ln 2)(x+1)}\,dx$$

$$= \frac{1}{\ln 2} \ln|x+1| \Big|_{0.3}^{1}$$

$$= \frac{1}{\ln 2}\left(\ln|1+1| - \ln|0.3+1|\right)$$

$$= 0.62$$

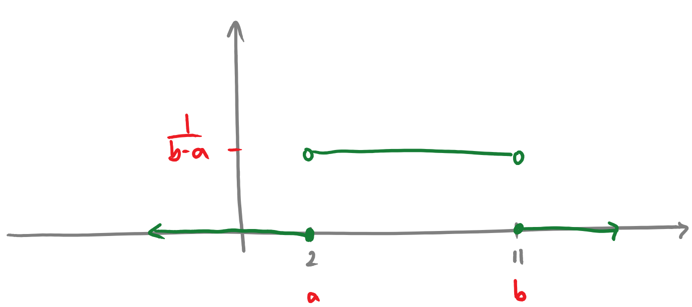
d) $$P(0.5 < X < 0.75) = \int_{0.5}^{0.75} \frac{1}{(\ln 2)(x+1)}\,dx$$

$$= \frac{1}{\ln 2} \ln|x+1| \Big|_{0.5}^{0.75}$$

$$= \frac{1}{\ln 2}\left(\ln|0.75+1| - \ln|0.5+1|\right)$$

$$= 0.22$$

4. uniform:




a) $$\frac{1}{b-a} = \frac{1}{11-2} = \frac{1}{9}$$ so $$f(x) = \begin{cases} \frac{1}{9} & 2 < x < 11 \\ 0 & \text{otherwise} \end{cases}$$

b) $P(3 < X < 8) = \int_3^8 f(x)\,dx$

$= \int_3^8 \frac{1}{9}\,dx$

$= \frac{1}{9}x \Big|_3^8$

$= \frac{1}{9}(8-3)$

$= \frac{5}{9} \approx 0.56$

c) $P(X > 7) = \int_7^\infty f(x)\,dx$

$= \int_7^{11} \frac{1}{9}\,dx$

$= \frac{1}{9}x \Big|_7^{11}$

$= \frac{1}{9}(11-7)$

$= \frac{4}{9} \approx 0.44$

d) $P(X < 4) = \int_{-\infty}^4 f(x)\,dx$

$= \int_2^4 \frac{1}{9}\,dx$

$= \frac{1}{9}x \Big|_2^4$

$= \frac{1}{9}(4-2)$

$= \frac{2}{9} \approx 0.22$

5. a) $P(X<0.1) = \int_{-\infty}^{0.1} f(x)\,dx$

$$= \int_{0}^{0.1} 2e^{-2x}\,dx$$

$$= 2\left(\frac{1}{-2}\right)e^{-2x}\Big|_0^{0.1}$$

$$= -e^{-2x}\Big|_0^{0.1}$$

$$= -e^{-2(0.1)} + e^{-2(0)}$$

$$= 0.18$$

b) $P(X\geq 0.1) = 1 - P(X<0.1)$

$$= 1 - 0.18$$

$$= 0.82$$

6. a) $P(2<X<5) = \int_{2}^{5} f(x)\,dx$

$$= \int_{2}^{5} 0.1e^{-0.1x}\,dx$$

$$= 0.1\left(\frac{1}{-0.1}\right)e^{-0.1x}\Big|_2^{5}$$

$$= -e^{-0.1x}\Big|_2^{5}$$

$$= -e^{-0.1(5)} + e^{-0.1(2)}$$

$$= 0.21$$

b) $\mu = \int_{-\infty}^{\infty} x \, f(x) \, dx$

$= \int_{0}^{\infty} x \cdot 0.1 e^{-0.1x} \, dx$

$= 0.1 \int_{0}^{\infty} x \, e^{-0.1x} \, dx$

$= 0.1 \, \dfrac{1}{0.1^2}$

$= 10$ months