Ch 27 Review

(3) $u = 0.2 \tan (3-2r)^{1/2}$

$u' = 0.2 \sec^2 (3-2r)^{1/2} \left[\frac{1}{2} (3-2r)^{-1/2} (-2) \right]$

$= \dfrac{-0.2 \sec^2 (3-2r)^{1/2}}{(3-2r)^{1/2}}$

or $\dfrac{-0.2 \sec^2 \sqrt{3-2r}}{\sqrt{3-2r}}$

(5) $y = \csc^2 (3x+2)$

$y = [\csc (3x+2)]^2$

$y' = 2[\csc (3x+2)][-\csc (3x+2) \cot (3x+2) (3)]$

$= -6 \csc^2 (3x+2) \cot (3x+2)$

(7) $y = 3\cos^4 x^2$

$y = 3[\cos x^2]^4$

$y' = 12[\cos x^2]^3[-\sin x^2 \ (2x)]$

$= -24x \sin x^2 \cos^3 x^2$

(9) $y = (e^{x-3})^2$

$= e^{2x-6}$

$y' = e^{2x-6}(2)$

$= 2e^{2x-6}$

(11) $y = 3\ln(x^2+1)$

$y' = 3\dfrac{1}{x^2+1}(2x)$

$= \dfrac{6x}{x^2+1}$

(13) $y = \cot^{-1}\left(\frac{x}{5}\right)$

$$y' = 10 \cdot \frac{1}{1+\left(\frac{x}{5}\right)^2} \left(\frac{1}{5}\right)$$

$$= \frac{2}{1+\frac{x^2}{25}}$$ Multiply by $\frac{25}{25}$

$$= \frac{50}{25+x^2}$$

(17) $y = (\csc 4x + \cot 4x)^{1/2}$

$$y' = \frac{1}{2}(\csc 4x + \cot 4x)^{-1/2}\left[-\csc 4x \cot 4x (4) - \csc^2 4x (4)\right]$$

$$= \frac{\frac{1}{2}\left[-4\csc 4x \cot 4x - 4\csc^2 4x\right]}{\sqrt{\csc 4x + \cot 4x}}$$

$$= \frac{-2\csc 4x\left[\cot 4x + \csc 4x\right]}{\sqrt{\csc 4x + \cot 4x}}$$

$$= (-2\csc 4x)\sqrt{\csc 4x + \cot 4x}$$

(27) $y = \ln^2(3+5\sin x)$

$y = [\ln(3+5\sin x)]^2$

$y' = 2[\ln(3+5\sin x)] \quad \frac{1}{3+5\sin x} \quad (\cos x)$

$= \frac{(2\cos x)\ln(3+5\sin x)}{3+5\sin x}$

(37) $\ln xy + ye^{-x} = 1$

Take $\frac{d}{dx}$: $\frac{1}{xy}\left[x\frac{dy}{dx} + y(1)\right] + y(e^{-x}(-1)) + e^{-x}\frac{dy}{dx} = 0$

$\frac{1}{y}\frac{dy}{dx} + \frac{1}{x} - ye^{-x} + e^{-x}\frac{dy}{dx} = 0$

$\left[\frac{1}{y} + e^{-x}\right]\frac{dy}{dx} = ye^{-x} - \frac{1}{x}$

$\frac{dy}{dx} = \frac{ye^{-x} - \frac{1}{x}}{\frac{1}{y} + e^{-x}}$ ✓

or $\frac{\left(ye^{-x} - \frac{1}{x}\right)(xy)}{\left(\frac{1}{y} + e^{-x}\right)(xy)}$

$= \frac{xy^2e^{-x} - y}{x + xye^{-x}}$ ✓

(39) $y = x\cos^{-1}x - \sqrt{1-x^2}$

$y' = x\left[\frac{-1}{\sqrt{1-x^2}}\right] + (\cos^{-1}x)(1) - \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= \frac{-x}{\sqrt{1-x^2}} + \cos^{-1}x + \frac{x}{\sqrt{1-x^2}}$

$= \cos^{-1}x$

(47) $y = e^{(x^2)}$

$y' = e^{(x^2)}(2x)$

$= 2xe^{x^2}$

$y'|_{x=0.5} \approx 1.2840$

$m_{tan} \approx 1.2840$

$m_{normal} \approx -0.7788$

$x_1 = 0.5 \rightarrow y = e^{(x^2)}$

$y_1 \approx 1.2840$

So $m \approx -0.7788$ $x_1 = 0.5$ $y_1 \approx 1.2840$

$y - y_1 = m(x - x_1)$

$y - 1.2840 = -0.7788(x - 0.5)$

$y - 1.2840 = -0.7788x + 0.3894$

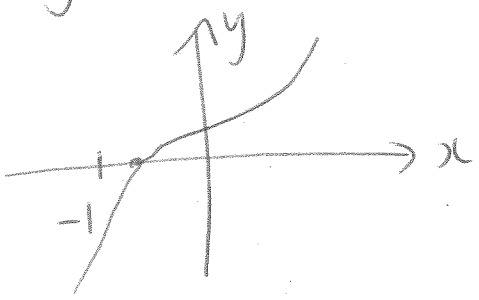
$0.7788x + y - 1.6734 = 0$

$\boxed{0.78x + y - 1.67 = 0}$

Any nonzero multiple of this is also acceptable.

(59) Use Wolfram Alpha to graph

$y = e^x - x^2$



$f(x) = e^x - x^2$
$f'(x) = e^x - 2x$

Choose $x_0 = -1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	
			-0.7331	(to 4)
-1	-0.6321	2.3679	-0.7038	(-0.70)
-0.7331	-0.0570	1.9466	-0.7035	(-0.70)
-0.7038	-0.0006	1.9023		

Table to 4 decimal places.
Final answer to 2 decimal places.

$x \approx -0.70$

(61) $y = e^x - 2e^{-x}$

$y' = e^x - 2e^{-x}(-1)$

$= e^x + 2e^{-x}$

$y'' = e^x + 2e^{-x}(-1)$

$= e^x - 2e^{-x}$

Set $y'' = 0$:

$e^x - 2e^{-x} = 0$

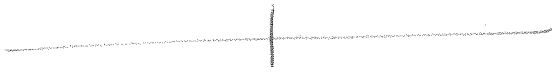
$e^x = 2e^{-x}$

$e^{2x} = 2$

$\ln(e^{2x}) = \ln 2$

$2x = \ln 2$

$x = \frac{\ln 2}{2}$



	⊖	$\frac{\ln 2}{2}$	⊕
y''			
y	Concave Down		Concave Up

y is concave up for $x > \frac{\ln 2}{2}$

(67) Motion is in vertical direction only.

We want $\left.\frac{dy}{dt}\right|_{t=5.0}$

$$y = 0.75 \sec \sqrt{0.15t} - 1$$

$$y = 0.75 \sec (0.15t)^{1/2} - 1$$

$$y' = 0.75 \sec (0.15t)^{1/2} \tan (0.15t)^{1/2} \tfrac{1}{2} (0.15t)^{-1/2} (0.15)$$

$$\left.y'\right|_{t=5.0} = \frac{0.75 \sec \sqrt{0.75} \tan \sqrt{0.75} \ (0.075)}{\sqrt{0.75}}$$

$$= \frac{0.75 \tan \sqrt{0.75} \ (0.075)}{\sqrt{0.75} \cos \sqrt{0.75}}$$

$$\approx 0.12 \ \frac{cm}{s}$$

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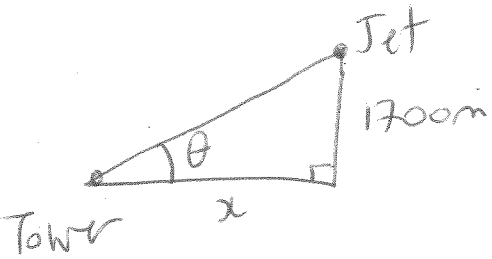


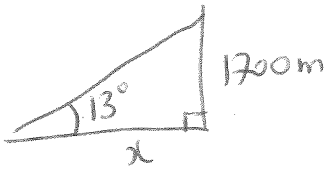
Find $\frac{d\theta}{dt}$ if $x = 220 \frac{m}{s}$ and $\theta = 13°$.

$$\theta = \tan^{-1} \frac{1700}{x}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx} \frac{dx}{dt}$$

$$= \frac{1}{1+\left(\frac{1700}{x}\right)^2} \left(-1700 x^{-2}\right) \frac{dx}{dt}$$

Missing Value: Find x



$$\tan 13° = \frac{1700}{x}$$

$$x \tan 13° = 1700$$

$$x = \frac{1700}{\tan 13°}$$

$$\frac{d\theta}{dt} = \frac{1}{1+(\tan 13°)^2} \frac{(-1700)}{\left(\frac{1700}{\tan 13°}\right)^2} (220)$$

$$\approx -0.0065 \frac{rad}{s}$$

Note: Units are $\frac{rad}{s}$, not $\frac{°}{s}$, because there are no loose degrees in $\frac{d\theta}{dt}$.