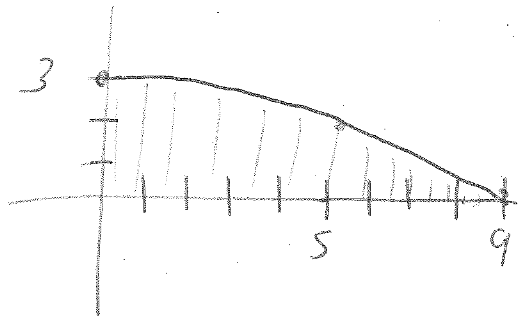
Ch 26 Review

(13) $y = \sqrt{9-x}$

x	y
0	$\sqrt{9} = 3$
5	$\sqrt{4} = 2$
9	0



$$A = \int_0^9 \sqrt{9-x}\, dx$$

$$\boxed{\begin{aligned} &\text{Sub } u = 9-x \\ &du = -dx \\ &-du = dx \\ &x=0 \Rightarrow u=9 \\ &x=9 \Rightarrow u=0 \end{aligned}}$$

$$= -\int_9^0 \sqrt{u}\, du$$

$$= -\frac{2}{3}\left[u^{3/2}\right]_9^0$$

$$= -\frac{2}{3}\left[0 - 27\right]$$

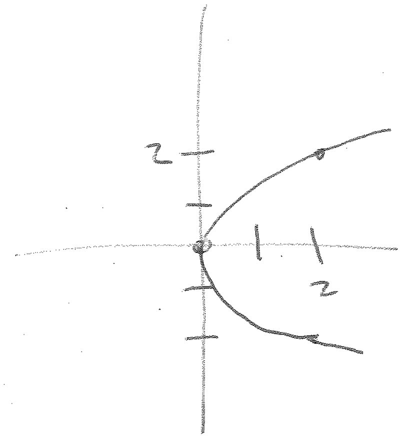
$$= 18$$

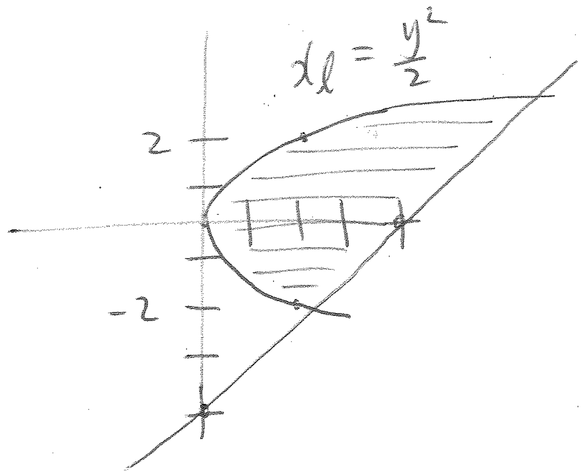
(15)

$$y^2 = 2x$$

$$x = \frac{y^2}{2}$$

y	x
-2	2
0	0
2	2





$$y = x - 4$$

$$y + 4 = x$$

$$x_r = y + 4$$

Intersection

$$x = x$$

$$\frac{y^2}{2} = y + 4$$

$$y^2 = 2y + 8$$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0$$

$$y = -2, 4$$

$\rightarrow$

15 Cont'd

$$A = \int_c^d (x_r - x_l)\,dy$$

$$= \int_{-2}^{4} \left[y + 4 - \frac{y^2}{2} \right] dy$$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^{4}$$

$$= \frac{40}{3} - \left(\frac{-14}{3} \right)$$

$$= \frac{54}{3}$$

$$= 18$$

(17) $y = x^3 - 2x^2$

x	y
0	0
1	-1
2	0
3	9

Intersection

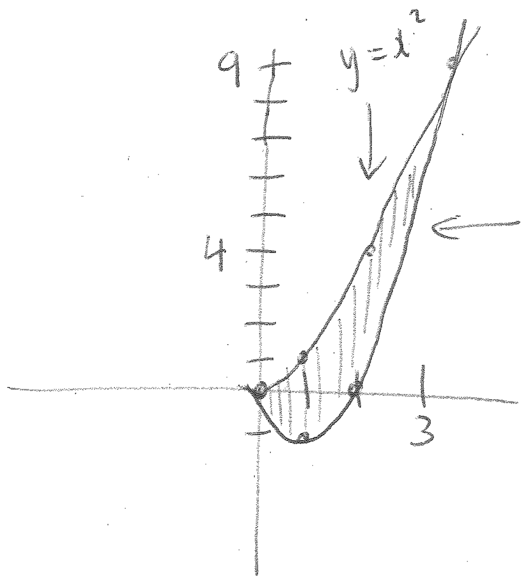
$y = y$

$x^3 - 2x^2 = x^2$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0, 3$



$$A = \int_0^3 (y_t - y_b)\,dx$$

$$= \int_0^3 (x^2 - x^3 + 2x^2)\,dx$$

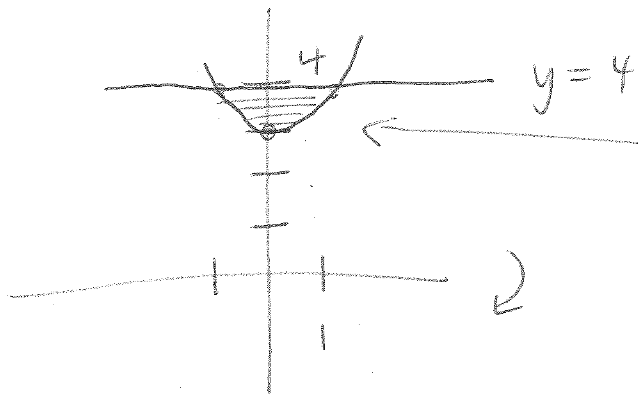
$$= \int_0^3 (3x^2 - x^3)\,dx$$

$$= \left[x^3 - \frac{x^4}{4}\right]_0^3$$

$$= 27 - \frac{81}{4}$$

$$= \frac{27}{4}$$

(21)



$y = 3 + x^2$

$y - 3 = x^2$

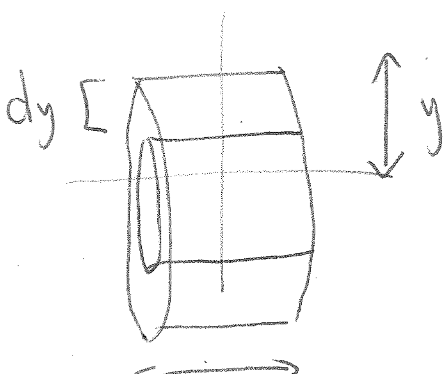
$x = \pm\sqrt{y-3}$

$x_r = \sqrt{y-3}$

$x_l = -\sqrt{y-3}$

Must use shells because region is not touching the x-axis.

$dV = 2\pi (\text{radius})(\text{height})(\text{thickness})$



$x_r - x_l$

$= 2\sqrt{y-3}$

$$dV = 2\pi y \left(2\sqrt{y-3}\right) dy$$

$$V = 4\pi \int_3^4 y\sqrt{y-3}\, dy$$

→

(21) Cont'd

Let $u = y - 3$

$du = dy$

$y = u + 3$

$y = 3 \Rightarrow u = 0$

$y = 4 \Rightarrow u = 1$

$$V = 4\pi \int_0^1 (u+3)\sqrt{u}\, du$$

$$= 4\pi \int_0^1 \left(u^{3/2} + 3u^{1/2}\right) du$$

$$= 4\pi \left[\frac{2}{5} u^{5/2} + 2u^{3/2} \right]_0^1$$

$$= 4\pi \left[\frac{2}{5} + 2 \right]$$

$$= 4\pi \left(\frac{12}{5} \right)$$

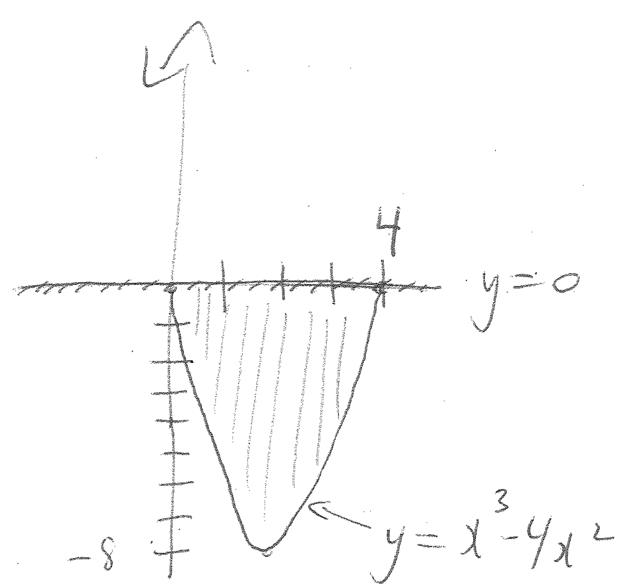
$$= \frac{48\pi}{5}$$

(23) Intersection of $y = x^3 - 4x^2$ and $y = 0$ (the x-axis) :

$$y = y$$
$$x^3 - 4x^2 = 0$$
$$x^2(x-4) = 0$$
$$x = 0, 4$$

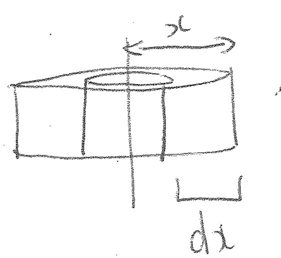
$y = x^3 - 4x^2$

x	y
0	0
2	-8
4	0



Must use shells because region is not touching the y-axis.

$dV = 2\pi(\text{radius})(\text{height})(\text{thickness})$



$y_t - y_b = -x^3 + 4x^2$

$\rightarrow$

23 Cont'd

$$dV = 2\pi x(-x^3+4x^2)\,dx$$

$$V = 2\pi \int_0^4 x(-x^3+4x^2)\,dx$$

$$= 2\pi \int_0^4 (-x^4+4x^3)\,dx$$

$$= 2\pi \left[-\frac{x^5}{5} + x^4 \right]_0^4$$

$$= 2\pi \left[-\frac{4^5}{5} + 256 \right]$$

$$= 2\pi \left(\frac{256}{5} \right)$$

$$= \frac{512\pi}{5}$$

(29)

$y^2 = x^3$

$y = x^{3/2}$

Intersection :

$y = y$

$x^{3/2} = 3x$

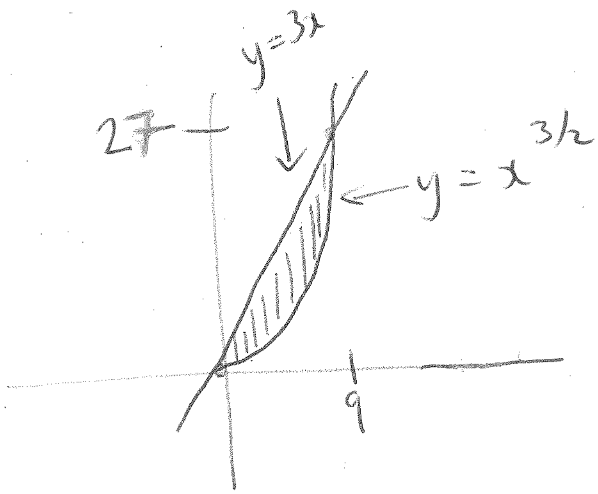
$x^{3/2} - 3x = 0$

$x(x^{1/2} - 3) = 0$

$x = 0 \qquad x^{1/2} - 3 = 0$

$x^{1/2} = 3$

$x = 9$



$$A = \int_0^9 (3x - x^{3/2})\,dx$$

$$= \left[\frac{3x^2}{2} - \frac{2}{5}x^{5/2}\right]_0^9 \qquad \rightarrow$$

(29) Cont'd $= \left[\frac{3}{2}(81) - \frac{2}{5}(243)\right]$

$= 24.3$

For a vertical slice:

$$x_e = x$$

$$y_e = \frac{y_b + y_t}{2}$$

$$= \frac{x^{3/2} + 3x}{2}$$

$$= \frac{x^{3/2}}{2} + \frac{3x}{2}$$

$$\int_A x_e \, dA = \int_0^9 x(3x - x^{3/2})\,dx$$

$$= \int_0^9 (3x^2 - x^{5/2})\,dx$$

$$= \left[x^3 - \frac{2}{7}x^{7/2}\right]_0^9$$

$$= \frac{729}{7}$$

$$\int_A y_e \, dA = \int_0^9 \left(\frac{x^{3/2}}{2} + \frac{3x}{2}\right)(3x - x^{3/2})\,dx$$

$$= \int_0^9 \left[\frac{3x^{5/2}}{2} - \frac{x^3}{2} + \frac{9x^2}{2} - \frac{3x^{5/2}}{2}\right]dx$$

$$= \left[-\frac{x^4}{8} + \frac{9x^3}{6}\right]_0^9$$

$$= \frac{2187}{8}$$

$$\bar{x} = \frac{729}{7}\left(\frac{1}{24.3}\right) = \frac{30}{7} \qquad \bar{y} = \frac{2187}{8}\left(\frac{1}{24.3}\right) = \frac{90}{8} = \frac{45}{4}$$

(43)

$$A = \int_0^3 (3x^2 - x^3)\,dx$$

$$= \left[x^3 - \frac{x^4}{4} \right]_0^3$$

$$= \frac{27}{4}$$

For a vertical slice, $x_e = x$

$$\int_A x_e\,dA = \int_0^3 x\,(3x^2 - x^3)\,dx$$

$$= \int_0^3 (3x^3 - x^4)\,dx$$

$$= \left[\frac{3x^4}{4} - \frac{x^5}{5} \right]_0^3$$

$$= \frac{243}{20}$$

$$\bar{x} = \frac{1}{A} \int_A x_e\,dA$$

$$= \left(\frac{4}{27}\right)\left(\frac{243}{20}\right)$$

$$= \frac{9}{5}\ \text{m}$$

$$= 1.8\ \text{m}$$