Section 26.6

5 $F = kx$

$\boxed{\text{amount of stretch beyond natural length}}$

The natural length is 8.0cm.

Sub $F = 6.0N$ and $x = 1.5\text{ cm}$:

$$6.0N = k(1.5\text{ cm})$$

$$k = 4.0\ \frac{N}{cm}$$

$$\boxed{F = 4.0x}$$

$$8.0\text{cm} \le \text{length} \le 10.0\text{cm}$$

$$0.0 \le x \le 2.0$$

$$W = \int_a^b F\,dx$$

$$= \int_{0.0}^{2.0} 4.0x\,dx$$

$$= \left[2.0x^2\right]_{0.0}^{2.0}$$

$$= 8.0\ N\cdot cm$$

$$\text{or}\quad 0.080\ N\cdot m$$

$$\text{or}\quad 0.080\ J$$

(11)

$$10 \le x \le 100$$

$$F = \frac{k}{x^2}$$

$$W = \int_a^b F\,dx$$

$$= \int_{10}^{100} k x^{-2}\,dx$$

$$= \left[-k x^{-1}\right]_{10}^{100}$$

$$= \frac{-k}{100} + \frac{k}{10}$$

$$= \frac{9k}{100} \text{ N.m}$$

or $0.09k$ N·m

(31)

$$y_{av} = \frac{1}{b-a}\int_a^b y\,dx$$

$$i_{av} = \frac{1}{b-a}\int_a^b i\,dt$$

$$= \frac{1}{4.0-0.0}\int_{0.0}^{4.0} (4t-t^2)\,dt$$

$$= \frac{1}{4.0}\left[2t^2 - \frac{t^3}{3}\right]_{0.0}^{4.0}$$

$$\approx 2.7\text{ A}$$

(35)

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = 0.04x^{3/2}$$

$$\frac{dy}{dx} = 0.06x^{1/2}$$

$$\left(\frac{dy}{dx}\right)^2 = 0.0036x$$

$$s = \int_0^{100} \sqrt{1 + 0.0036x}\, dx$$

Sub $u = 1 + 0.0036x$

$du = 0.0036\,dx$

$\dfrac{du}{0.0036} = dx$

$x = 0 \Rightarrow u = 1$

$x = 100 \Rightarrow u = 1.36$

$$= \frac{1}{0.0036}\int_1^{1.36} u^{1/2}\, du$$

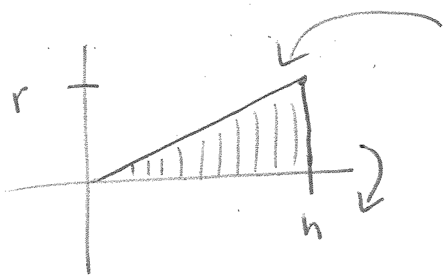
$$= \frac{1}{0.0036}\left[\frac{2}{3}u^{3/2}\right]_1^{1.36}$$

$$= \frac{1}{0.0036}\left(\frac{2}{3}\right)\left(1.36^{3/2} - 1\right)$$

$$\approx 109\text{m}$$

(37) $$S = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

(for revolution about x-axis)



$$y = mx + b$$

$$y = \frac{r}{h}x + b$$

$$y = \frac{r}{h}x$$

$$\frac{dy}{dx} = \frac{r}{h}$$

$$S = 2\pi \int_0^h \frac{r}{h}x\sqrt{1+\frac{r^2}{h^2}}\,dx$$

$$= 2\pi \frac{r}{h}\sqrt{1+\frac{r^2}{h^2}} \int_0^h x\,dx$$

$$= 2\pi \frac{r}{h}\sqrt{1+\frac{r^2}{h^2}}\left[\frac{x^2}{2}\right]_0^h$$

$$= 2\pi \frac{r}{h}\sqrt{1+\frac{r^2}{h^2}}\left(\frac{h^2}{2}\right)$$

$$= \pi r h\sqrt{1+\frac{r^2}{h^2}}$$

$$= \pi r\sqrt{h^2}\sqrt{1+\frac{r^2}{h^2}}$$

$$= \pi r\sqrt{h^2+r^2} \quad \text{or} \quad \pi r\sqrt{r^2+h^2}$$