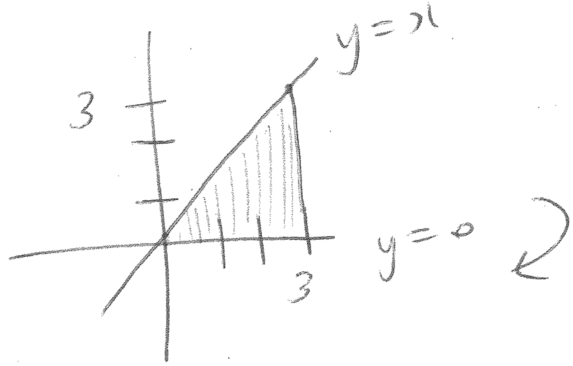
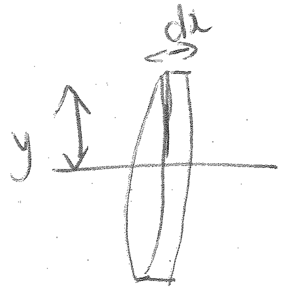
Section 26.3

7

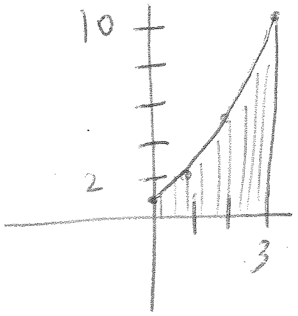




$$dV = \pi (\text{radius})^2 (\text{thickness})$$
$$= \pi y^2 dx$$

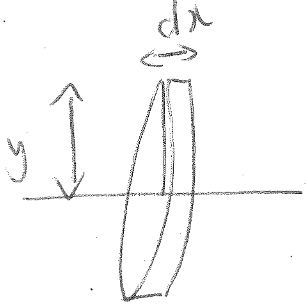
$$V = \pi \int_0^3 y^2 dx$$
$$= \pi \int_0^3 x^2 dx$$
$$= \pi \left[\frac{x^3}{3} \right]_0^3$$
$$= 9\pi$$

(13)



$y = x^2 + 1$

$y = 0$

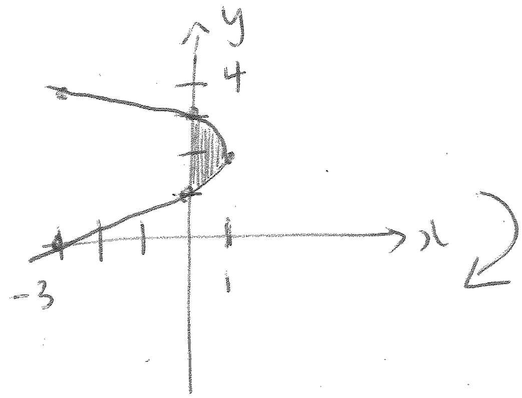


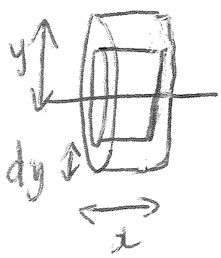
$$dV = \pi (\text{radius})^2 (\text{thickness})$$
$$= \pi y^2 \, dx$$

$$V = \pi \int_0^3 y^2 \, dx$$
$$= \pi \int_0^3 (x^2+1)^2 \, dx$$
$$= \pi \int_0^3 (x^4 + 2x^2 + 1) \, dx$$
$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_0^3$$
$$= \frac{348\pi}{5}$$

(15) Graph $x = 4y - y^2 - 3$

y	x
0	-3
1	0
2	1
3	0
4	-3





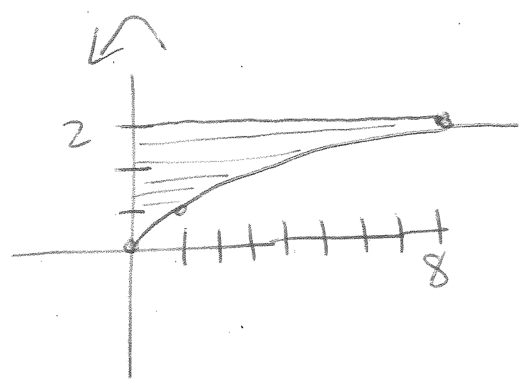
$dV = 2\pi(\text{radius})(\text{height})(\text{thickness})$
$= 2\pi\, y\, x\, dy$

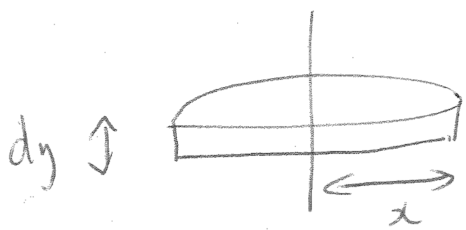
$$V = 2\pi \int_1^3 y\,x\,dy$$
$$= 2\pi \int_1^3 y(4y - y^2 - 3)\,dy$$
$$= 2\pi \int_1^3 (4y^2 - y^3 - 3y)\,dy$$
$$= 2\pi \left[\frac{4y^3}{3} - \frac{y^4}{4} - \frac{3y^2}{2} \right]_1^3$$
$$= 2\pi \left[2.25 - \left(\frac{-2.5}{6} \right) \right]$$
$$= \frac{16\pi}{3}$$

(17)

$$y = x^{1/3}$$

x	y
0	0
1	1
8	2





$$dV = \pi(\text{radius})^2(\text{thickness})$$
$$= \pi x^2\,dy$$

$$V = \pi \int_0^2 x^2\,dy$$
$$= \pi \int_0^2 y^6\,dy$$
$$= \pi \left[\frac{y^7}{7}\right]_0^2$$
$$= \frac{128\pi}{7}$$

$$\boxed{\begin{aligned} x^{1/3} &= y \\ x &= y^3 \\ x^2 &= y^6 \end{aligned}}$$

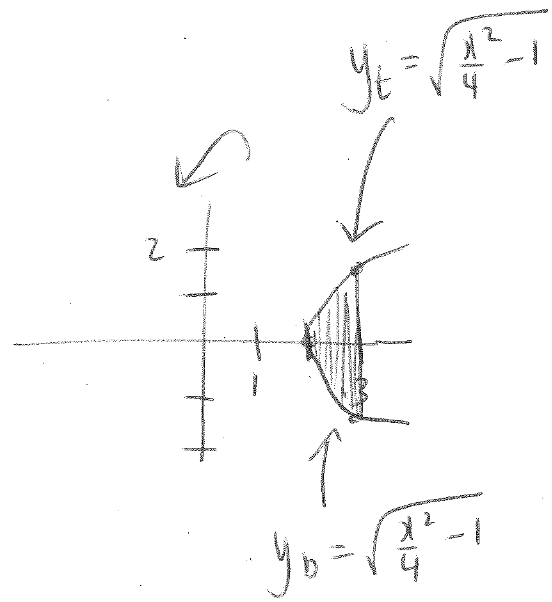
(21) $x^2 - 4y^2 = 4$

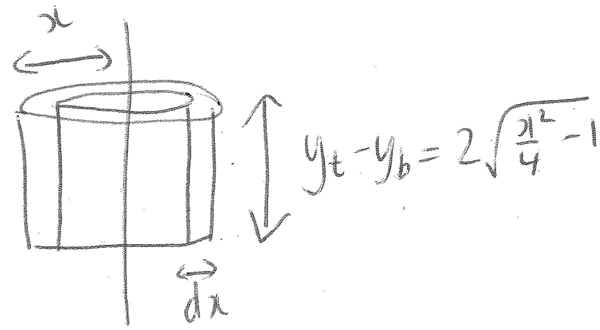
$$-4y^2 = -x^2 + 4$$

$$y^2 = \frac{x^2}{4} - 1$$

$$y = \pm\sqrt{\frac{x^2}{4} - 1}$$

x	y
2	0
3	± 1.1





$$dV = 2\pi(\text{radius})(\text{height})(\text{thickness})$$

$$= 2\pi\, x \left(2\sqrt{\frac{x^2}{4} - 1}\right) dx$$

$$V = 4\pi \int_2^3 x\left(\frac{x^2}{4} - 1\right)^{1/2} dx$$

$\rightarrow$

(21) Cont'd

$$V = 8\pi \int_0^{5/4} u^{1/2}\, du$$

$$= 8\pi \left[\frac{2}{3} u^{3/2} \right]_0^{5/4}$$

$$= \frac{16\pi}{3} \left(\frac{\sqrt{5}}{2} \right)^3$$

$$= \frac{16\pi}{3} \left(\frac{5\sqrt{5}}{8} \right)$$

$$= \frac{10\pi\sqrt{5}}{3}$$

Sub $u = \frac{x^2}{4} - 1$

$du = \frac{x}{2}\, dx$

$2\, du = x\, dx$

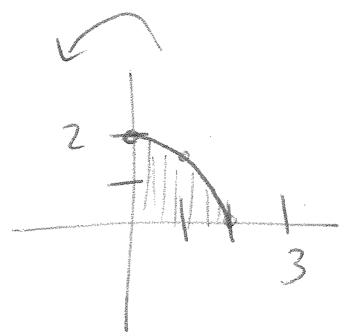
$x = 2 \Rightarrow u = 0$

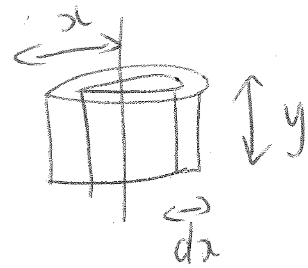
$x = 3 \Rightarrow u = \frac{5}{4}$

(25)

$y = \sqrt{4-x^2}$

x	y
0	2
1	$\sqrt{3}$
2	0





$$dV = 2\pi(\text{radius})(\text{height})(\text{thickness})$$

$$= 2\pi\, x\, y\, dx$$

$$V = 2\pi \int_0^2 x\, y\, dx$$

$$= 2\pi \int_0^2 x\sqrt{4-x^2}\, dx$$

Sub $u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$x=0 \Rightarrow u=4$

$x=2 \Rightarrow u=0$

$$= -\frac{1}{2}\cdot 2\pi \int_4^0 u^{1/2}\, du$$

$\rightarrow$

(25) Cont'd

$$= -\pi \int_4^0 u^{1/2}\,du$$

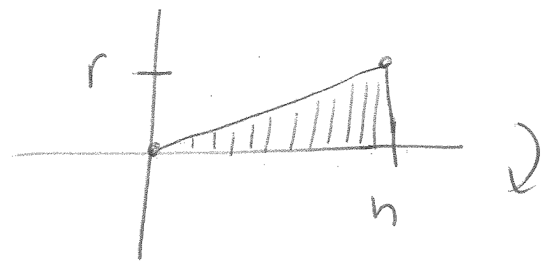
$$= -\pi \left[\frac{2}{3} u^{3/2} \right]_4^0$$

$$= -\pi \left[0 - \frac{16}{3} \right]$$

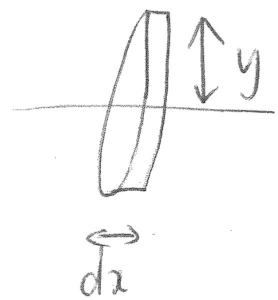
$$= \frac{16\pi}{3}$$

(31) $y = \frac{r}{h} x$

x	y
0	0
h	r

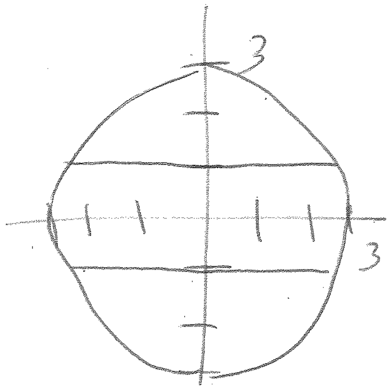


DISK METHOD

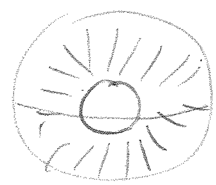


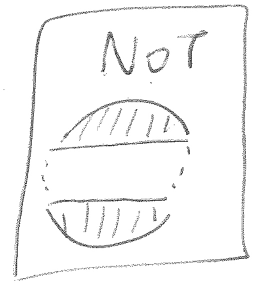
$$dV = \pi(\text{radius})^2(\text{thickness})$$
$$= \pi y^2 \, dx$$
$$V = \pi \int_0^h y^2 \, dx$$
$$= \pi \int_0^h \frac{r^2}{h^2} x^2 \, dx$$
$$= \frac{\pi r^2}{h^2} \left[\frac{x^3}{3} \right]_0^h$$
$$= \frac{\pi r^2}{h^2} \left(\frac{h^3}{3} \right)$$
$$= \frac{1}{3} \pi r^2 h$$

(37)

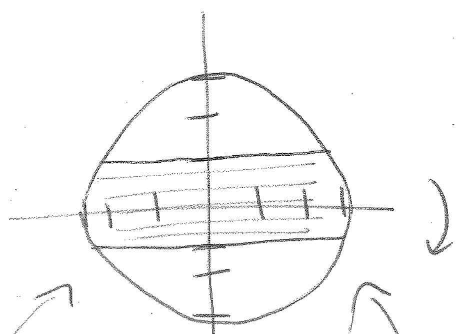


Side View of Sphere With a Hole in Centre

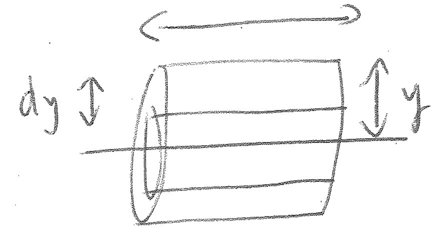




Cannot use Disk Method
Must use Shell Method.



$x_r - x_l = 2\sqrt{9-y^2}$



Circle of radius 3

$$x^2+y^2=9$$

$$x^2=9-y^2$$

$$x=\pm\sqrt{9-y^2}$$

$x_l = -\sqrt{9-y^2}$ $\qquad$ $x_r = \sqrt{9-y^2}$

$$dV = 2\pi\,(\text{radius})(\text{height})(\text{thickness})$$
$$= 2\pi\left(2\sqrt{9-y^2}\right) y\,dy$$

$\rightarrow$

(37) Cont'd

$$dV = 4\pi y (9-y^2)^{1/2} \, dy$$

$$V = 4\pi \int_0^1 y (9-y^2)^{1/2} \, dy$$

$$\boxed{\begin{aligned} u &= 9-y^2 \\ du &= -2y\,dy \\ -\tfrac{1}{2}\,du &= y\,dy \\ y=0 &\Rightarrow u=9 \\ y=1 &\Rightarrow u=8 \end{aligned}}$$

$$= -\frac{1}{2} \cdot 4\pi \int_9^8 u^{1/2} \, du$$

$$= -2\pi \left[\frac{2}{3} u^{3/2} \right]_9^8$$

$$= -\frac{4\pi}{3} \left[8^{3/2} - 27 \right]$$

$$\approx 18.3 \text{ cm}^3$$