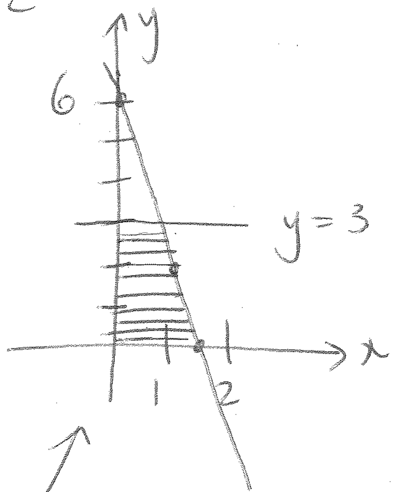
26.2

5

Table of Values for $y = 6 - 4x$

x	y
0	6
1	2



$x_l = 0$

$y = 6 - 4x$

$4x = 6 - y$

$x = 1.5 - 0.25y$

$x_r = 1.5 - 0.25y$

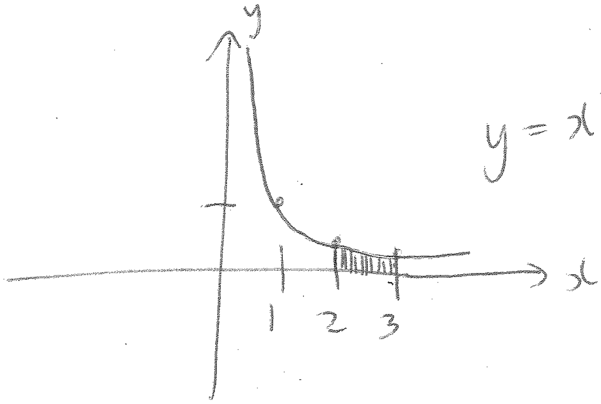
$$A = \int_0^3 (x_r - x_l)\,dy$$

$$= \int_0^3 (1.5 - 0.25y)\,dy$$

$$= \left[1.5y - 0.125y^2\right]_0^3$$

$$= 3.375 \text{ or } \frac{27}{8}$$

(9)



$y = x^{-2}$ or $y = \frac{1}{x^2}$

x	$y = \frac{1}{x^2}$
1	1
2	1/4
3	1/9

$$A = \int_2^3 (y_t - y_b)\,dx$$

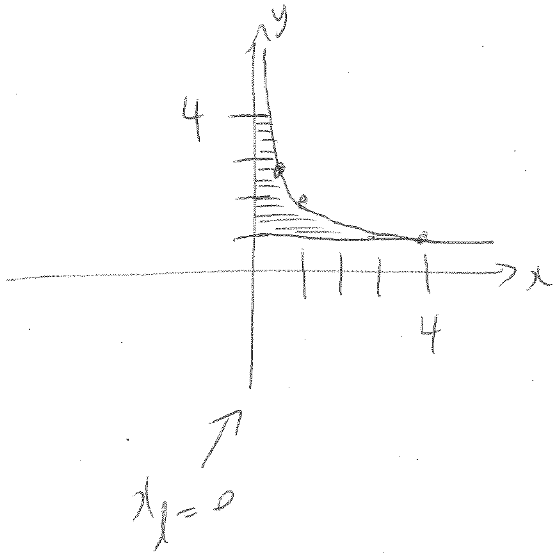
$$= \int_2^3 (x^{-2} - 0)\,dx$$

$$= \left[-x^{-1}\right]_2^3$$

$$= -\frac{1}{3} + \frac{1}{2}$$

$$= \frac{1}{6}$$

(13)



$y = \frac{2}{\sqrt{x}}$

$\sqrt{x} = \frac{2}{y}$

$x = \frac{4}{y^2}$

$x_r = \frac{4}{y^2}$ or $x_r = 4y^{-2}$

Table of Values for $y = \frac{2}{\sqrt{x}}$

x	y
0.5	2.8
1	2
4	1

$$A = \int_1^4 (x_r - x_l)\,dy$$

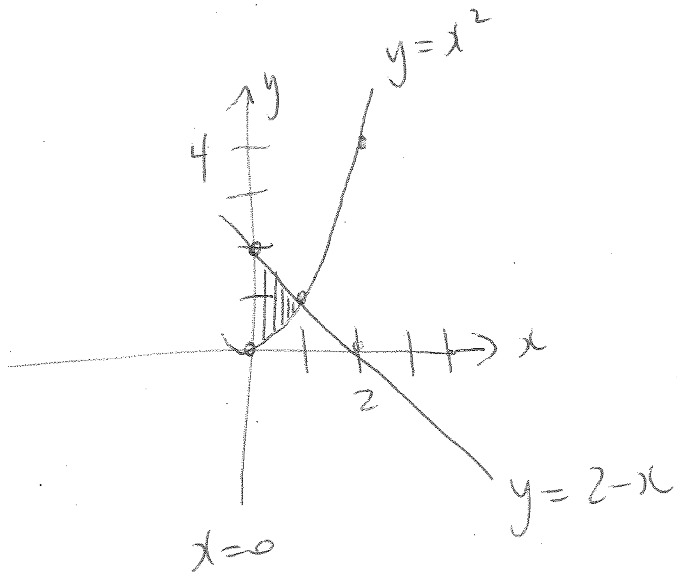
$$= \int_1^4 4y^{-2}\,dy$$

$$= \left[-4y^{-1}\right]_1^4$$

$$= -\frac{4}{4} + \frac{4}{1}$$

$$= 3$$

(19)



Want the area with $x \geq 0$.

From the graph we can see that the curves intersect at $x = 1$. The algebra:

$$y = y$$
$$x^2 = 2 - x$$
$$x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$
$$x = -2, 1$$

$$A = \int_0^1 (y_t - y_b)\,dx$$
$$= \int_0^1 (2 - x - x^2)\,dx$$
$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$
$$= 2 - \frac{1}{2} - \frac{1}{3}$$
$$= \frac{7}{6}$$

(23) Intersection:

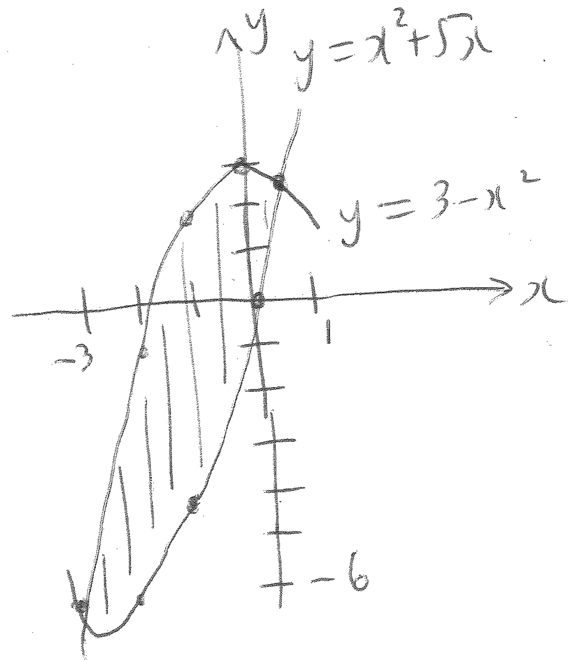
$$y = y$$
$$x^2 + 5x = 3 - x^2$$
$$2x^2 + 5x - 3 = 0$$

Quadratic Formula $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-5 \pm \sqrt{49}}{4}$$
$$= \frac{-5 \pm 7}{4}$$
$$= -3, \tfrac{1}{2}$$

x	$y = x^2 + 5x$
-3	-6
-2	-6
-1	-4
0	0
0.5	2.75

x	$y = 3 - x^2$
-3	-6
-2	-1
-1	2
0	3
0.5	2.75



→

(23) Cont'd

$$A = \int_{-3}^{0.5} (y_t - y_b)\,dx$$

$$= \int_{-3}^{0.5} (3 - x^2 - x^2 - 5x)\,dx$$

$$= \int_{-3}^{0.5} (3 - 2x^2 - 5x)\,dx$$

$$= \left[3x - \frac{2x^3}{3} - \frac{5x^2}{2} \right]_{-3}^{0.5}$$

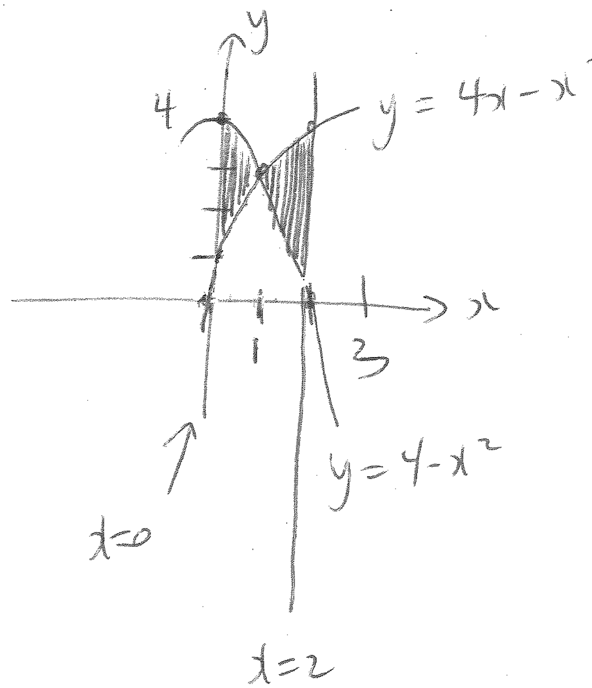
$$\approx 0.7917 + 13.5$$

$$\approx 14.2917 \quad \text{or} \quad \frac{343}{24}$$

(29)

x	$y = 4 - x^2$
0	4
1	3
2	0

x	$y = 4x - x^2$
0	0
1	3
2	4



Want the sum of two areas.

$$A_1 = \int_0^1 (y_t - y_b)\,dx$$

$$= \int_0^1 (4 - x^2 - 4x + x^2)\,dx$$

$$= \int_0^1 (4 - 4x)\,dx$$

$$= \left[4x - 2x^2\right]_0^1$$

$$= 2$$

→

(29) Cont'd

$$A_2 = \int_1^2 (y_t - y_b)\,dx$$

$$= \int_1^2 (4x - x^2 - 4 + x^2)\,dx$$

$$= \int_1^2 (4x - 4)\,dx$$

$$= \left[2x^2 - 4x\right]_1^2$$

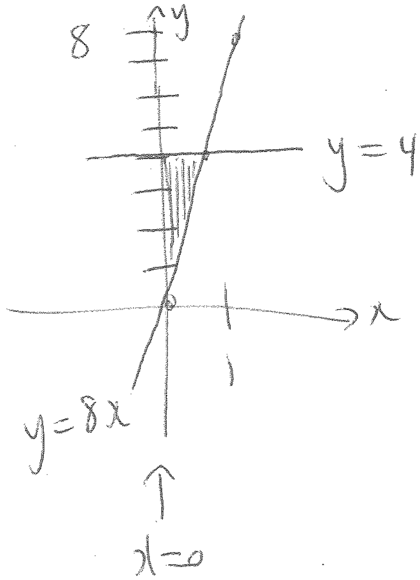
$$= 0 - (-2)$$

$$= 2$$

Total area $= A_1 + A_2$

$= 4$

(39)



Find area by:

a) vertical slices

b) horizontal slices

a) Intersection: $y = y$

$$8x = 4$$

$$x = 0.5$$

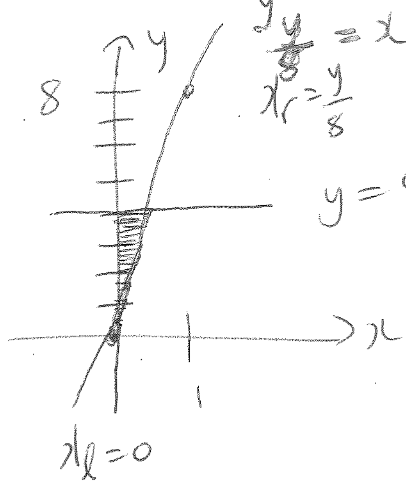
$$A = \int_0^{0.5} (y_t - y_b)\,dx$$

$$= \int_0^{0.5} (4 - 8x)\,dx$$

$$= \left[4x - 4x^2\right]_0^{0.5}$$

$$= 1$$

b)



$y = 8x$

$\frac{y}{8} = x$

$x_r = \frac{y}{8}$

→

39 Cont'd

$$A = \int_0^4 (x_r - x_l)\,dy$$

$$= \int_0^4 \left(\frac{y}{8} - 0\right) dy$$

$$= \left[\frac{y^2}{16}\right]_0^4$$

$$= 1$$

(43) Power is the rate of change of work.
Power is the derivative of work.
Work is the integral of power.

$$W = \int_0^3 P\,dt$$
$$= \int_0^3 (12t - 4t^2)\,dt$$
$$= \left[6t^2 - \frac{4t^3}{3}\right]_0^3$$
$$= 18 \text{ J}$$

(45) Displacement is the integral of velocity.

Change in displacement from $t=10$ to $t=100$

is $\displaystyle\int_{10}^{100} v\,dt$

$$= \int_{10}^{100} [1-0.01(2t+1)^{1/2}]\,dt$$

Sub $u = 2t+1$
$du = 2dt$
$\frac{du}{2} = dt$
$t=10 \Rightarrow u=21$
$t=100 \Rightarrow u=201$

$$= \frac{1}{2}\int_{21}^{201} [1-0.01u^{1/2}]\,du$$

$$= \frac{1}{2}\left[u - 0.01\left(\frac{2}{3}u^{3/2}\right)\right]_{21}^{201}$$

$$= \frac{1}{2}[182.002 - 20.358]$$

$$\approx 80.8 \text{ km}$$