Ch 25 Review

5 $\displaystyle\int_1^4 \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}} \right) dx$

$= \displaystyle\int_1^4 \left(\frac{x^{1/2}}{2} + 2x^{-1/2} \right) dx$

$= \left[\frac{1}{2} \cdot \frac{2}{3} x^{3/2} + 2 \cdot 2 x^{1/2} \right]_1^4$

$= \left(\frac{8}{3} + 8 \right) - \left(\frac{1}{3} + 4 \right)$

$= \frac{7}{3} + 4$

$= \frac{19}{3}$

13 $\int \frac{dn}{(9-5n)^3}$

$$\boxed{\begin{aligned} u &= 9-5n \\ du &= -5\,dn \\ -\frac{du}{5} &= dn \end{aligned}}$$

$= \frac{-1}{5}\int \frac{du}{u^3}$

$= \frac{-1}{5}\int u^{-3}\,du$

$= -\frac{1}{5}\left(-\frac{1}{2}u^{-2}\right) + C$

$= \frac{1}{10}(9-5n)^{-2} + C$

(17) $$\int_0^2 \frac{3x\,dx}{\sqrt[3]{1+2x^2}}$$

$$\boxed{\begin{aligned} u &= 1+2x^2 \\ du &= 4x\,dx \\ \frac{du}{4} &= x\,dx \\ x=0 &\Rightarrow u=1 \\ x=2 &\Rightarrow u=9 \end{aligned}}$$

$$= \frac{3}{4}\int_1^9 \frac{du}{\sqrt[3]{u}}$$

$$= \frac{3}{4}\int_1^9 u^{-1/3}\,du$$

$$= \frac{3}{4}\left[\frac{3}{2}u^{2/3}\right]_1^9$$

$$= \frac{9}{8}\left[u^{2/3}\right]_1^9$$

$$= \frac{9}{8}\left[9^{2/3}-1\right]$$

(19) $\int x^2 (1-2x^3)^4 \, dx$

$$\boxed{\begin{aligned} u &= 1-2x^3 \\ du &= -6x^2\,dx \\ -\frac{du}{6} &= x^2\,dx \end{aligned}}$$

$$= -\frac{1}{6}\int u^4 \, du$$

$$= -\frac{1}{6}\left[\frac{1}{5}u^5\right] + C$$

$$= -\frac{1}{30}(1-2x^3)^5 + C$$

(25) Find y given $\frac{dy}{dx} = 3 - x^2$ and $(x, y) = (-1, 3)$.

$$\frac{dy}{dx} = 3 - x^2$$

$$dy = (3 - x^2)\,dx$$

$$\int dy = \int (3 - x^2)\,dx$$

$$\boxed{y = 3x - \frac{x^3}{3} + C}$$

Sub $y = 3$, $x = -1$:

$$3 = -3 + \frac{1}{3} + C$$

$$6 - \frac{1}{3} = C$$

$$\frac{17}{3} = C$$

$$\boxed{y = 3x - \frac{x^3}{3} + \frac{17}{3}}$$

(31) $f''(x) = \dfrac{1}{\sqrt{6x+5}}$

$f''(x) = (6x+5)^{-1/2}$

$f'(x) = \int (6x+5)^{-1/2}\,dx$

$$\boxed{\begin{aligned} u &= 6x+5 \\ du &= 6\,dx \\ \frac{du}{6} &= dx \end{aligned}}$$

$= \frac{1}{6}\int u^{-1/2}\,du$

$= \frac{1}{6}\left(2u^{1/2}\right) + C_1$

$= \frac{1}{3}(6x+5)^{1/2} + C_1$

$f(x) = \int \left[\frac{1}{3}(6x+5)^{1/2} + C_1\right]dx$

Same substitution as above

$= \frac{1}{6}\int \left[\frac{1}{3}u^{1/2} + C_1\right]du$

$= \frac{1}{6}\left[\frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C_1 u\right] + C_2$

$= \frac{1}{27}(6x+5)^{3/2} + C_1\,\frac{6x+5}{6} + C_2$

Also acceptable:

$f(x) = \frac{1}{27}(6x+5)^{3/2} + C_1 x + C_2$

(45) Approximate $\int_1^4 x\sqrt[3]{2x^2+1}\,dx$

using the Trapezoidal Rule with $n=3$.

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{3} = 1$$

x	$y = x\cdot\sqrt[3]{2x^2+1}$
1	1.4422
2	4.1602
3	8.0052
4	12.8301

$$\int_1^4 x\cdot\sqrt[3]{2x^2+1}\,dx \approx \frac{1}{2}\left[1.4422 + 2(4.1602 + 8.0052) + 12.8301\right]$$

$$\approx 19.30$$

(47) Approximate $\int_1^4 x \cdot \sqrt[3]{2x^2+1}\,dx$ using Simpson's Rule with $n=6$.

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{6} = 0.5$$

x	$y = x \cdot \sqrt[3]{2x^2+1}$
1	1.4422
1.5	2.6478
2	4.1602
2.5	5.9528
3	8.0052
3.5	10.3018
4	12.8301

$$\int_1^4 x \cdot \sqrt[3]{2x^2+1}\,dx \approx \frac{0.5}{3}\big[1.4422 + 4(2.6478) + 2(4.1602)$$

$$+4(5.9528) + 2(8.0052) + 4(10.3018) + 12.8301\big]$$

$$\approx 19.04$$